

A calibration test of Stewart platform

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Abstract—A whole calibration test process of Stewart platform including the error analysis, calibration model, measure process, data analysis, parameter errors determination and the iterative result analysis is presented.

Index Terms—calibration, test, Stewart platform

I. INTRODUCTION

PARALLEL mechanisms have been applied in many fields. Before the machines being put into use, the most important step is calibration of the parallel mechanisms. Because of the errors in design, manufacture and assembly, the real machine has many differences from the theoretical prototype. The task of calibration is to find what the real parameters of the machine are. The calibration process has many problems and attracts many researchers' attentions. Maurine and Dombre[1], Zhuang[2], Masory[3], Vischer and Clavel[4] calibrate parallel mechanisms by measuring all the information of the moving platform. Goswami[5], Daney and Emir[6], Chen[7] discussed the calibration method using part information of the upper platform. Everett[8], Judd and Knasinski[9], Mooring and Padavala[10], Wang and Masory[11], Chai[12] discussed the calibration model. Nahvi[13], Wampler[14], Zhuang[15], Khalil and Besnard[16], Maurine[17], Abdul[18] did research jobs on the measurement of the parallel mechanisms. In fact, the calibration of the parallel mechanism is a systematic work including error analysis, calibration model set up, data measurement, data analysis and precision determination, parameters calculation and iterative result analysis and test. A total calibration procedure is presented in this paper.

II. ERROR ANALYSIS

Error analysis, the base for the calibration, includes two parts, the theoretical analysis and the real manipulator structure analysis. Firstly, the theoretical analysis procedure takes all the possible errors into thinking. No only the joints coordinates and the original length of the prismatic joints, but also the errors of

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the parts such as the axes' uncoaxiality etc. The all parameter error analysis is carried out by setting up the D-H parameter model of the whole mechanism. The influence of each parameter's error to the position and pose of the moving platform can be found. The analysis result shows that compare to the end errors brought by the joint coordinates and poles' lengths, the errors brought by the errors of the parts are extraordinary small. The conclusion can be drawn that the main error facts of the Stewart platform is the joint coordinates and the poles' length.

The second step of the error analysis is real manipulator structure analysis. The real manipulator is a 6-UCU parallel mechanism. According to the assembly situation, biggest errors are the poles' length, and then the coordinates of the base joints, the smallest are the upper joints.

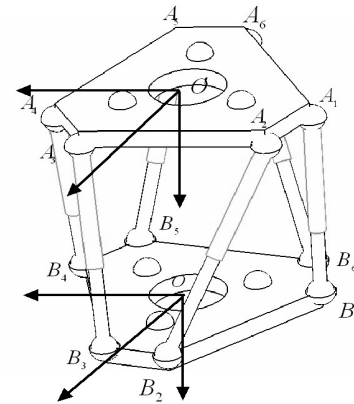


Fig.1 The Stewart platform

III. THE CALIBRATION MODEL

Shown in Fig.1 is the prototype of the Stewart platform. The base coordinate system is O-XYZ and the moving one is O'-xyz. The coordinate of the joint A_i in the moving system is $[x_{ai} \ y_{ai} \ z_{ai}]^T$. The coordinate of the joint B_i in base system is $[x_{bi} \ y_{bi} \ z_{bi}]^T$. The translation matrix is

$$R = R(\psi, y)R(\theta, z)R(\varphi, x) = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \quad (1)$$

Where

$$\begin{aligned} q_{11} &= \cos \psi \cos \theta \\ q_{12} &= -\cos \psi \sin \theta \cos \varphi + \sin \psi \sin \varphi \\ q_{13} &= \cos \psi \sin \theta \sin \varphi + \sin \psi \cos \varphi \end{aligned}$$

$$\begin{aligned}
q_{21} &= \sin \theta \\
q_{22} &= \cos \theta \cos \varphi \\
q_{23} &= -\cos \theta \sin \varphi \\
q_{31} &= -\sin \psi \cos \theta \\
q_{32} &= \sin \psi \sin \theta \cos \varphi + \cos \psi \sin \varphi \\
q_{33} &= -\sin \psi \sin \theta \sin \varphi + \cos \psi \cos \varphi
\end{aligned}$$

The inverse kinematics of the mechanism is

$$(l_i + l_{i0})^2 = (x + x_i)^2 + (y + y_i)^2 + (z + z_i)^2 \quad (2)$$

Where

$$\begin{aligned}
x_i &= q_{11}x_{ai} + q_{12}y_{ai} + q_{13}z_{ai} - x_{bi} \\
y_i &= q_{21}x_{ai} + q_{22}y_{ai} + q_{23}z_{ai} - y_{bi} \\
z_i &= q_{31}x_{ai} + q_{32}y_{ai} + q_{33}z_{ai} - z_{bi}
\end{aligned}$$

The complete differential of eq.2 is

$$\begin{aligned}
&(l_i + l_{i0})(dl_i + dl_{i0}) \\
&= (x + x_i)(dx + dx_i) + (y + y_i)(dy + dy_i) + (z + z_i)(dz + dz_i)
\end{aligned} \quad (3)$$

where

$$\begin{aligned}
dx_i &= d(q_{11}x_{ai} + q_{12}y_{ai} + q_{13}z_{ai}) - dx_{bi} \\
dy_i &= d(q_{21}x_{ai} + q_{22}y_{ai} + q_{23}z_{ai}) - dy_{bi} \\
dz_i &= d(q_{31}x_{ai} + q_{32}y_{ai} + q_{33}z_{ai}) - dz_{bi}
\end{aligned}$$

Let

$$\begin{aligned}
L_i &= l_i + l_{i0} \\
a_i &= x + x_i \\
b_i &= y + y_i \\
c_i &= z + z_i
\end{aligned}$$

$$\begin{aligned}
d_i &= a_i(-\sin \psi \cos \theta x_{ai} + \sin \psi \sin \theta \cos \varphi y_{ai} + \cos \psi \sin \varphi y_{ai} \\
&\quad - \sin \psi \sin \theta \sin \varphi z_{ai} + \cos \psi \cos \varphi z_{ai}) \\
&+ c_i(-\cos \psi \cos \theta x_{ai} + \cos \psi \sin \theta \cos \varphi y_{ai} - \sin \psi \sin \varphi y_{ai} \\
&\quad - \cos \psi \sin \theta \sin \varphi z_{ai} - \sin \psi \cos \varphi z_{ai})
\end{aligned}$$

$$\begin{aligned}
e_i &= a_i(-\cos \psi \sin \theta x_{ai} - \cos \psi \cos \theta \cos \varphi y_{ai} \\
&\quad + \cos \psi \cos \theta \sin \varphi z_{ai}) \\
&+ b_i(\cos \theta x_{ai} - \sin \theta \cos \varphi y_{ai} + \sin \theta \sin \varphi z_{ai}) \\
&+ c_i(\sin \psi \sin \theta x_{ai} + \sin \psi \cos \theta \cos \varphi y_{ai} \\
&\quad - \sin \psi \cos \theta \sin \varphi z_{ai})
\end{aligned}$$

$$\begin{aligned}
f_i &= a_i(\cos \psi \sin \theta \sin \varphi y_{ai} + \sin \psi \cos \varphi y_{ai} \\
&\quad + \cos \psi \sin \theta \cos \varphi z_{ai} - \sin \psi \sin \varphi z_{ai}) \\
&+ b_i(-\cos \theta \sin \varphi y_{ai} - \cos \theta \cos \varphi z_{ai}) \\
&+ c_i(-\sin \psi \sin \theta \sin \varphi y_{ai} + \cos \psi \cos \varphi y_{ai} \\
&\quad - \sin \psi \sin \theta \cos \varphi z_{ai} - \cos \psi \sin \varphi z_{ai})
\end{aligned}$$

$$\begin{aligned}
m_i &= a_i q_{11} + b_i q_{21} + c_i q_{31} \\
n_i &= a_i q_{12} + b_i q_{22} + c_i q_{32} \\
s_i &= a_i q_{13} + b_i q_{23} + c_i q_{33} \\
k_i &= a_i q_{110} + b_i q_{210} + c_i q_{310} \\
t_i &= a_i q_{120} + b_i q_{220} + c_i q_{320} \\
h_i &= a_i q_{130} + b_i q_{230} + c_i q_{330}
\end{aligned}$$

Eq.3 can be rewritten as

$$Adl + Adl_0 = BdX + CdP_1 - DdP_2 - EdP_3 \quad (4)$$

where

$$A = \begin{bmatrix} L_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_6 \end{bmatrix}$$

$$B = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 & f_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 & f_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 & f_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 & f_4 \\ a_5 & b_5 & c_5 & d_5 & e_5 & f_5 \\ a_6 & b_6 & c_6 & d_6 & e_6 & f_6 \end{bmatrix}$$

$$C = \begin{bmatrix} m_1 & n_1 & s_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & n_2 & s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_3 & n_3 & s_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_4 & n_4 & s_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_5 & n_5 & s_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_6 & n_6 & s_6 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} k_1 & t_1 & h_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_2 & t_2 & h_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_3 & t_3 & h_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_4 & t_4 & h_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_5 & t_5 & h_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_6 & t_6 & h_6 & 0 & 0 \end{bmatrix}$$

$$dP_1 = [dx_{a1} \ dy_{a1} \ dz_{a1} \ dx_{a2} \ dy_{a2} \ dz_{a2} \ dx_{a3} \ dy_{a3} \ dz_{a3} \ dx_{a4} \ dy_{a4} \ dz_{a4} \ dx_{a5} \ dy_{a5} \ dz_{a5} \ dx_{a6} \ dy_{a6} \ dz_{a6}]^T$$

$$dP_2 = [dx_{b1} \ dy_{b1} \ dz_{b1} \ dx_{b2} \ dy_{b2} \ dz_{b2} \ dx_{b3} \ dy_{b3} \ dz_{b3} \ dx_{b4} \ dy_{b4} \ dz_{b4} \ dx_{b5} \ dy_{b5} \ dz_{b5} \ dx_{b6} \ dy_{b6} \ dz_{b6}]^T$$

$$dP_3 = [d\psi_0 \ d\theta_0 \ d\varphi_0]^T$$

$$dl = [dl_1 \ dl_2 \ dl_3 \ dl_4 \ dl_5 \ dl_6]^T$$

$$dl_0 = [dl_{10} \ dl_{20} \ dl_{30} \ dl_{40} \ dl_{50} \ dl_{60}]^T$$

$$dX = [dx \ dy \ dz]^T$$

The 42 parameters' errors calculated by Eq.4 can not satisfy the Eq.2. Take these values as the start values of the parameters, another value searching procedure based on the inverse kinematics should be carried. The searching result will be the calibration result. The theoretical parameters of the parallel mechanism are listed in Table I where the coordinates of the joint have been changed into the base system and the vector l_0 is the original length of all prismatic poles.

Table I
THEORETICAL PARAMETERS

parameter	value
A_1	[1127.7234,-451.66359162545, 130]
A_2	[1127.7234,113.24849332075,456.15214431216]
A_3	[1127.7234,338.4150983047,326.15214431216]
A_4	[1127.7234,338.4150983047,-326.15214431216]
A_5	[1127.7234,113.24849332075,-456.15214431216]
A_6	[1127.7234,113.24849332075,-456.15214431216]
B_1	[0,-784.21447755781,908.29931916121]
B_2	[0,-394.50304585482 1133.29931916121]
B_3	[0,1178.71752341263,225]
B_4	[0,1178.71752341263,-225]
B_5	[0,-394.50304585481,-1133.29931916122]
B_6	[0,-784.21447755781,-908.29931916121]
l_0	[1410,1410,1410,1410,1410,1410]

IV. MEASUREMENT

The data measurement is one important step in the calibration process. In this test, the measure equipment is a camera system named K600 Roydom system which can measure the 6D data of the object.



Fig.2 The measurement equipment

In the test, ten configurations are selected to be measured. The errors of the platform can be calculated by analyzing the measured configurations and the given configurations. The precision of the platform can be determined. Listed in Table II are the errors of the platform. Table III shows the 6-dimensional errors and the resultant translational error and rotation error which are the precisions of the parallel of the mechanism before calibration.

Table II
ERRORS OF THE PLATFORM

	Dx(mm)	Dy(mm)	Dz(mm)	Drx(°)	Dry(°)	Drz(°)
1	2.827514	-0.280167	3.709829	-0.277281	-0.023706	0.1577563
2	1.751953	0.5917206	2.956422	-0.138582	-0.014554	0.1131055
3	1.336059	-2.139175	3.419505	-0.200882	0.0511657	0.1171841
4	1.086425	-0.305802	4.707778	-0.214291	0.0690886	0.1669263
5	1.968994	-1.410651	1.599746	-0.137012	-0.059070	0.0535857
6	1.633667	-1.061482	2.081010	-0.054584	0.0935578	0.0776155
7	1.275146	-0.509939	4.171698	-0.310075	-0.087930	0.0722073
8	1.393554	-0.077879	3.610021	-0.243773	0.1472015	0.2512802
9	1.183593	-2.757204	4.904428	-0.174829	-0.162393	0.3825359
10	0.726318	-0.033307	1.946974	-0.105294	0.12402	-0.197893

Table III
THE PRECISION OF THE PLATFORM

motions	errors
X translation(mm)	2.8275146
Y translation(mm)	2.7572041
Z translation(mm)	4.9044285
X rotation(°)	0.3100758
Y rotation(°)	0.1623933
Z rotation(°)	0.3825359
Translational error(mm)	5.7494769
Rotational error(°)	0.4508555

V. PARAMETER ERROR DETERMINATION

Take the measurement data into the calibration model, the parameters' errors can be found. The errors of the upper joints, base joints and the poles length are dA , dB and dl_0 respectively.

$$dA = \begin{bmatrix} -5.1067009 & -6.7557487 & -6.0164909 & -3.8974724 & -3.3941765 & -4.4349480 \\ -0.5473154 & -1.1176873 & 1.8618792 & 2.6301289 & -0.9011956 & -2.1035898 \\ 1.4887085 & 0.4510967 & 1.6483012 & -1.5964214 & -2.4228985 & -1.6202698 \end{bmatrix}$$

$$dB = \begin{bmatrix} 5.9580736 & 3.6633029 & 2.1217835 & 3.5012929 & 2.5103045 & 3.8282938 \\ -0.6779175 & 0.0182741 & -0.5967959 & -1.1734561 & 1.8977907 & -1.4382324 \\ -1.5344876 & 0.8458225 & 3.1549988 & -3.1325073 & 0.1073026 & 2.7606840 \end{bmatrix}$$

$$dl_0 = [-2.5316770 \ -1.5290976 \ 0.1485070 \ -0.8412749 \ 0.0290310 \ -0.5454170]$$

VI. CALIBRATION RESULT TEST AND ANALYSIS

Taking the 42 parameters' errors into the control model, measure the configurations of the moving platform again. Use the same measurement method and data analysis procedure, the calibration result can be analyzed. Shown in Fig.3 to Fig.10 are the compare results of the 6-dimensional errors and the resultant translational and rotation errors.

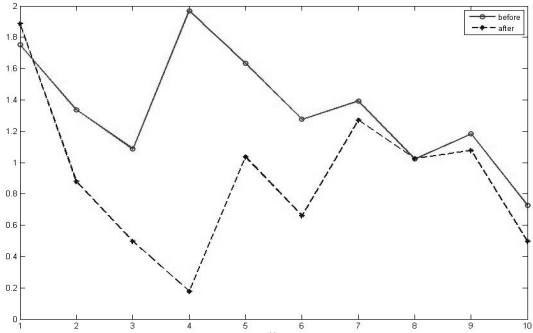


Fig.3 X translational errors

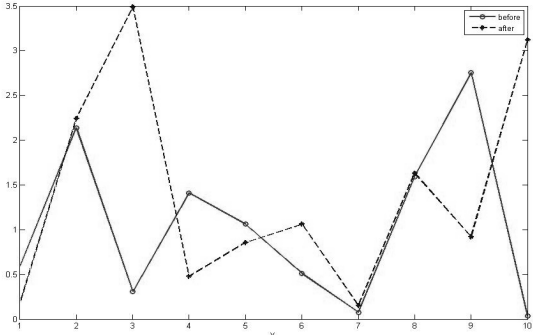


Fig.4 Y translation errors

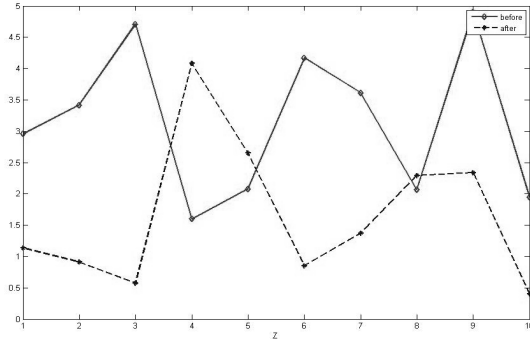


Fig.5 Z translational errors

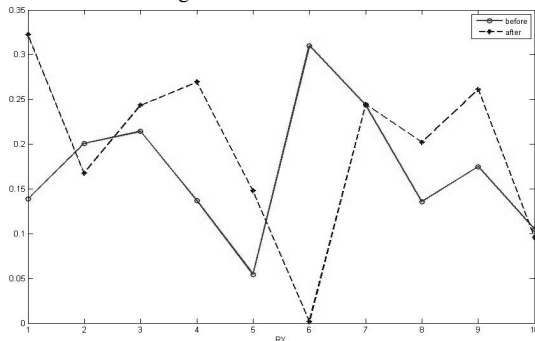


Fig.6 X rotational errors

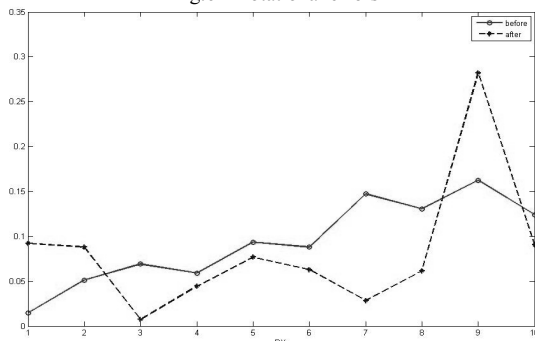


Fig.7 Y rotational errors

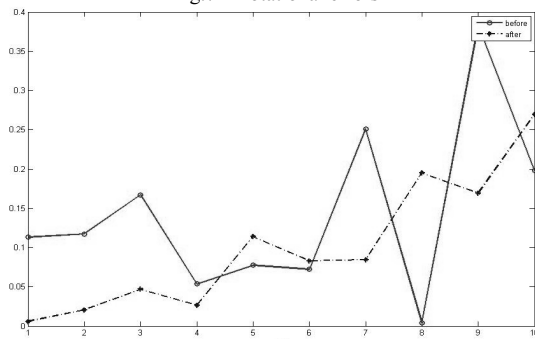


Fig.8 Z rotational errors

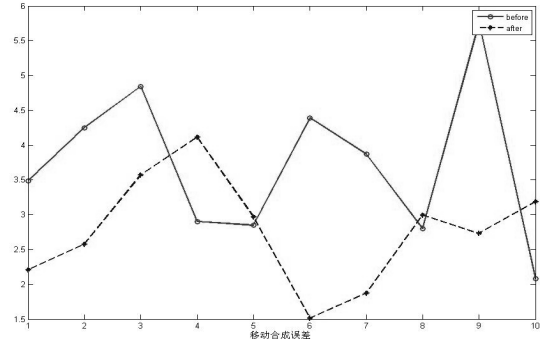


Fig.9 The resultant translational errors

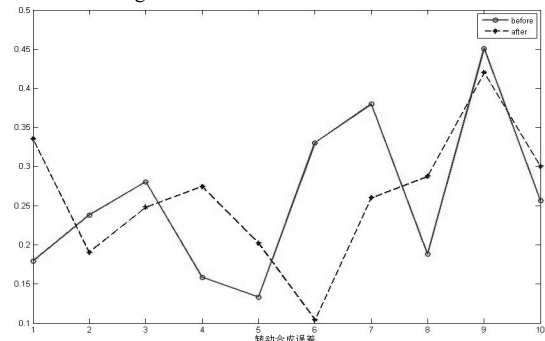


Fig.10 The resultant rotational errors

Listed in Table IV are the precision of the parallel mechanism after calibration. From the figures and data, it can be seen that:

- (1) The X translational errors are smaller in most configurations;
- (2) Some of the Y translational errors become bigger and some are smaller, but the biggest value increases;
- (3) Most of the Z translational errors are smaller, but the biggest values is almost the same as the value before;
- (4) Most of the rotation errors around X are bigger;
- (5) Most of the rotation errors around Y are smaller, but the biggest values is bigger than the before value;
- (6) The rotation errors around Z are smaller in most configurations;
- (7) Both of the resultant translational errors and the resultant rotational errors are smaller in most configurations;
- (8) Half of the 6 dimensional precision increase and the resultant precision also increase;
- (9) There are still 3 dimensional errors become bigger. That because this is the result after the first round of calibration test, the errors can be debased after several times of test.

Table IV
COMPARE RESULTS

motions	before	after	Compare results
X translation(mm)	2.8275146	1.8856576	-0.941857
Y translation(mm)	2.7572041	3.49075053	+0.73354643
Z translation(mm)	4.9044285	4.0845612	-0.8198673
X rotation(°)	0.3100758	0.3223551	+0.0122799
Y rotation(°)	0.1623933	0.28206449	+0.11967119
Z rotation(°)	0.3825359	0.26951893	-0.11301697
Translation(mm)	5.7494769	4.11599448	-1.63348242
Rotation(°)	0.4508555	0.41999784	-0.03085766

VII. CONCLUSION

A whole calibration test of parallel platform is introduced. The test shows that the precision can be increased by the method. There are still some errors become bigger, they can be corrected by doing the calibration test repeatedly.

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