

Theoretical and experimental investigation of the effect of oil aeration on the load-carrying capacity of a hydrodynamic journal bearing

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Abstract: It is widely assumed that the presence of air bubbles in the lubricating oil of a hydrodynamic bearing gives rise to a reduced load-carrying capacity, because of the high compressibility and low viscosity of the air and its tendency, therefore, to upset the hydrodynamic effect. The aim of the work described in the current paper was to investigate the accuracy of this assumption by theoretical and experimental means, and also to provide quantitative data relating to the concentration of air bubbles and their size that are required for any discernible effect.

The paper has the following three main contributions: (a) a theoretical model based on Reynolds equation, but modified to allow for the effect of aeration on lubricant viscosity and density, is proposed; (b) a novel method of injecting air bubbles into lubricating oil and for measuring their size and concentration was developed; and (c) an experimental hydrodynamic bearing test rig was implemented and run with both aerated and non-aerated lubricating oil, and in each case measurements of the load-carrying capacity for various operating speeds were made.

The results from both theoretical and experiment work show that the presence of air bubbles in the lubricating oil leads to a slight decrease in bearing load-carrying capacity at high operating speeds. For normal operating speeds, however, (i.e. those resulting in eccentricity ratios greater than 0.6) results show that the presence of air bubbles has little effect on bearing load-carrying capacity.

Keywords: hydrodynamic, journal bearing, air bubbles, lubricant, Reynolds equation, lubricant aeration

1 INTRODUCTION

Oil lubricated hydrodynamic journal bearings are used extensively in many types of machinery. It is widely assumed that the presence of air bubbles within the lubricating oil is detrimental to the operation of the bearing, and often manufacturers and machine operators go to considerable lengths to remove any air bubbles. An example of this is in the power generation industry where hydrodynamic bearings are used to support large generator and turbine rotors, and where costly de-aerating equipment is provided to treat the oil before recirculation.

Despite the above, there have been relatively few published studies concerned with the effect of air bubbles on hydrodynamic bearing performance. Theoretical work by Abdel-Latif *et al.* [1] considered the effect of aerated oil in circular thrust bearings. Later Chamniprasart *et al.* [2] developed a theoretical model for journal bearings lubricated by aerated oil, based on a modified Reynolds equation, and Nikolajsen [3, 4] developed a theoretical model based on viscosity and density values that were modified to account for the presence of air bubbles.

Practical measurements of the effect of lubricant aeration on bearing performance are particularly scarce. Experimental measurements of the effect of aeration on oil viscosity were reported by Calderwood *et al.* [5] and by Hayward [6], showing that viscosity increases with increasing bubble density. Roach and Goodwin [7] reported observations of increased

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stability of a journal bearing when aerated oil was used as the lubricant. Then in 1997 An *et al.* [8] measured a slight decrease in load capacity when aerated oil was used with a journal bearing, and later Diaz and San Andres [9] observed an increase in damping when aerated oil was used to lubricate squeeze-film damper bearings. This was also confirmed by experimental work published by Nikolajsen *et al.* [10] which showed that aeration resulted in an increase in both oil film stiffness and damping.

From the above it can be seen that there is a need for more quantitative information about the effect of air bubbles in the lubricant of journal bearings. Work described in the current paper was therefore undertaken in order to:

- devise a means through which the number and size of the air bubbles within a lubricant could be both controlled and measured,
- enable measurements to be made of how bearing load-carrying capacity is affected by lubricant aeration, and
- investigate whether the practical observations could be predicted by theory.

2 THEORETICAL MODEL BASED ON REYNOLDS EQUATION

The variation of lubricant pressure in a journal bearing is represented by Reynolds equation

$$\frac{\partial}{\partial \phi} \left(\frac{\bar{\rho} \bar{h}^3}{\bar{\eta}} \frac{\partial \bar{p}}{\partial \phi} \right) + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left(\frac{\bar{\rho} \bar{h}^3}{\bar{\eta}} \frac{\partial \bar{p}}{\partial \bar{z}} \right) = \Gamma \frac{\partial(\bar{\rho} \bar{h})}{\partial \phi} \quad (1)$$

Normally, Reynolds equation in the above form is solved numerically to yield the lubricant pressure throughout the bearing oil film, and from this the load-carrying capacity can be obtained. In the case of bubbly oil, however, the density of the air–oil lubricant, in dimensionless form, can be defined as [11]

$$\begin{aligned} \bar{\rho} &= \frac{\rho}{\rho_{\text{oil}}} = \frac{m_{\text{oil}} + m_{\text{air}}}{\rho_{\text{oil}}(V_{\text{oil}} + V_{\text{air}})} \\ &= \frac{1 + m_{\text{air}}/m_{\text{oil}}}{\rho_{\text{oil}}(V_{\text{oil}}/m_{\text{oil}} + [V_{\text{air}}/m_{\text{air}}](m_{\text{air}}/m_{\text{oil}})]} \\ &= \frac{1 + \mu}{1 + \mu \rho_{\text{oil}} RT / p_{\text{air}}} = \frac{(1 + \mu) \bar{p}_{\text{air}}}{\bar{p}_{\text{air}} + \mu} \end{aligned} \quad (2)$$

where

$$\bar{p}_{\text{air}} = \frac{p_{\text{air}}}{\rho_{\text{oil}} RT} \quad (3)$$

For air bubbles in oil, the relationship between the air pressure and the surrounding oil pressure was

shown by Smith [12] to be

$$\bar{p}_{\text{air}} = \bar{p}_{\text{oil}} + \frac{2\bar{\sigma}}{\bar{r}} \quad (4)$$

where $\bar{r} = r/c$. So that the non-dimensional density of the air–oil mixture is then given by

$$\bar{\rho} = \frac{(1 + \mu)(\bar{p}_{\text{oil}} + 2\bar{\sigma}/\bar{r})}{\mu + \bar{p}_{\text{oil}} + 2\bar{\sigma}/\bar{r}} \quad (5)$$

The bubble radius in equation (5) is obtained from the equation of state

$$p_{\text{air}} V_{\text{air}}^\gamma = (p_{\text{air}})_{\text{in}} (V_{\text{air}})_{\text{in}}^\gamma \quad (6)$$

where the suffix ‘in’ indicates reference to conditions at the oil inlet to the bearing.

Substituting $\gamma = 1$, $V = (4/3)\pi r^3$, and \bar{p}_{air} from above gives

$$\bar{p}_{\text{oil}} \bar{r}^3 + 2\bar{\sigma} \bar{r}^2 - \left[(p_{\text{oil}})_{\text{in}} + \frac{2\bar{\sigma}}{\bar{r}_{\text{in}}} \right] \bar{r}_{\text{in}}^3 = 0 \quad (7)$$

which can be solved to give three roots, the correct value of \bar{r} for use in equation (5) being that between 0 and \bar{r}_{in} [11].

In addition to the above effect of the air bubbles on the lubricant density, there is also an effect on the lubricant viscosity. On considering the fact that the rotating journal is in effect shearing both oil and air, and for the purposes of modelling considering all of the air placed as a ‘block’ adjacent to all of the oil as in Fig. 1, then the shear force supported by the oil viscosity is given by

$$F = \frac{\eta_{\text{oil}} A_{\text{oil}} U}{c} \quad (8)$$

where U is the journal surface velocity, and A is the cross sectional area of the oil ‘block’. If the air viscosity is negligible compared with that of the oil then this is also the shear force supported by the whole air/oil

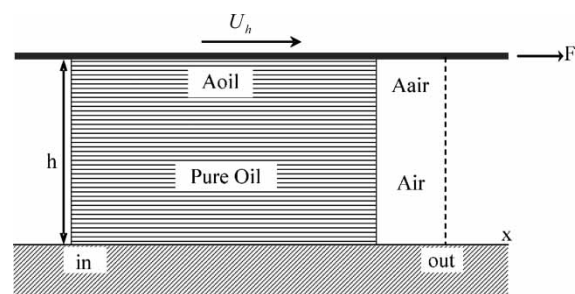


Fig. 1 Air and oil mix lubricating a bearing, showing the air and the oil separately for analysis purposes

mix, so that the effective viscosity of the mixture is then given by

$$\begin{aligned}\eta_1 &= \frac{F/A_{\text{total}}}{U/c} = \frac{\eta_{\text{oil}}A_{\text{oil}}U/c(A_{\text{oil}} + A_{\text{air}})}{U/c} \\ &= \frac{\eta_{\text{oil}}A_{\text{oil}}}{A_{\text{oil}} + A_{\text{air}}} = \frac{\eta_{\text{oil}}}{1 + (A_{\text{air}}/A_{\text{oil}})} = \frac{\eta_{\text{oil}}}{1 + (V_{\text{air}}/V_{\text{oil}})}\end{aligned}\quad (9)$$

The overall dimensionless dynamic viscosity for the air and oil together is then given by

$$\begin{aligned}\bar{\eta}_1 &= \frac{\eta_1}{\eta_{\text{oil}}} = \frac{1}{1 + (V_{\text{air}}/V_{\text{oil}})} \\ &= \frac{1}{1 + (m_{\text{air}}RT/p_{\text{air}})/(m_{\text{oil}}/\rho_{\text{oil}})} = \frac{p_{\text{air}}}{p_{\text{air}} + \mu RT \rho_{\text{oil}}} \\ &= \frac{\bar{p}_{\text{air}}}{\bar{p}_{\text{air}} + \mu} = \frac{\bar{p}_{\text{oil}} + 2\bar{\sigma}/\bar{r}}{\bar{p}_{\text{oil}} + 2\bar{\sigma}/\bar{r} + \mu} = \frac{\bar{\rho}}{1 + \mu}\end{aligned}\quad (10)$$

in which η_1 is less than η_{oil} . Equation (10) does not allow for the effect of bubble surface tension however, which tends to increase the net viscosity. Work by Dong [11], omitted here for the sake of brevity, has shown the added viscosity due to surface tension, in dimensionless form, to be

$$\bar{\eta}_2 = \frac{\eta_2}{\eta_{\text{oil}}} = \frac{\sigma}{2d^2r\eta_{\text{oil}}} \frac{dA/dt}{[(\partial v_x/\partial y)^2 + (\partial v_z/\partial y)^2]}\quad (11)$$

It is noteworthy that this expression, which allows for bubble distortion due to lubricant flow in both the circumferential and axial directions, is an extension of previous work published by Nikolajsen [4] which allowed only for unidirectional flow.

The overall effective dynamic viscosity, allowing for both the lower viscosity of the air and also the tendency of surface tension effects to increase effective viscosity, is given by

$$\bar{\eta} = \bar{\eta}_1 + \bar{\eta}_2\quad (12)$$

Equations (5) and (12) were substituted in to equation (1) which was then solved using over-relaxation for a finite difference grid representing the oil film, to yield the oil film pressure profile. Numerical integration then yielded the oil film forces. This theory was represented in the form of a computer program, and was run for various journal eccentricities and attitude angles to determine the steady-state locus of the journal in the bearing. All of this is described in more detail by Dong [11].

3 A NOVEL TECHNIQUE FOR AERATING THE OIL AND FOR MEASURING LEVEL OF AERATION

Preliminary tests to identify a means of creating air bubbles within the oil were based on use of a

pressurized air supply fed into a porous stone that was submerged within the oil supply tank. The method is also described by Roach and Goodwin [7]. This method tended to result in relatively large air bubbles being created however, and these tended not to readily remain in suspension, and it was not easy to control their size. Therefore, a different technique was developed in which a centrifugal pump is submerged within the oil tank, and a pressurized air supply is fed into the pump impeller, as shown in Fig. 2. The action of the impeller on the air and oil resulted in much smaller air bubbles being created in the oil, and the oil took on a milky white appearance. The efficiency of the process was improved by incorporating a boost pump to feed the oil into the impeller, thus preventing the impeller from stalling while facilitating rapid aeration of the tank contents. The rating of the aerator pump was 110 l/min at 2800 r/min, and the boost pump rating was 8 l/min at a head of 2 m. The air injection nozzle feeding the impeller had a diameter of 0.5 mm and the air flowrate injected was 50 ml/min. These aerator system settings were established by trial and error with a view to maximizing the number of air bubbles suspended in the oil, and minimizing the size of the air bubbles; theoretical work by Nikolajsen [4] and Dong [11] had predicted that these conditions would have the largest effect on the operation of the bearing. These conditions were also considered to be more representative of industrial machinery.

The level of oil aeration was measured as follows. A sample of oil was placed on a microscope slide, and a cover slip applied to the sample. The distance between the cover slip and the base slide was maintained at 0.1 mm by spacer material located around the oil sample and between the cover and slide, as shown in Fig. 3.

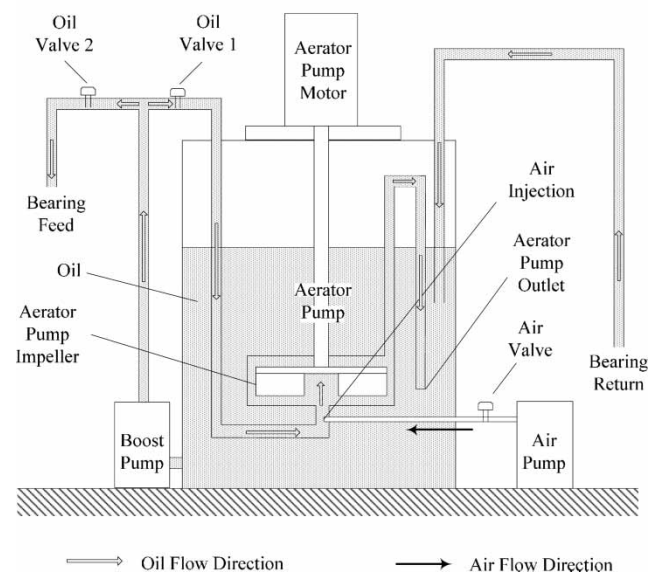


Fig. 2 Schematic diagram of oil aerator

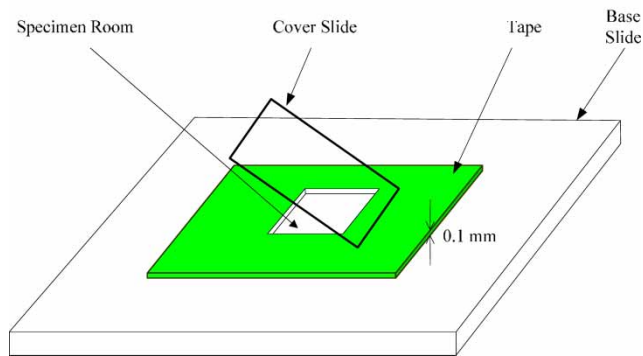


Fig. 3 Experimental set-up for bubble measurement

The oil sample was then placed under a microscope and photographed at 25 times magnification, as shown in Fig. 4(a). Once photographed, the number and size of the bubbles within an area of 4.47 mm^2 of the photographed sample were determined using image analysis software. The oil aeration level was then expressed in terms of the mean bubble radius r_m and the mean

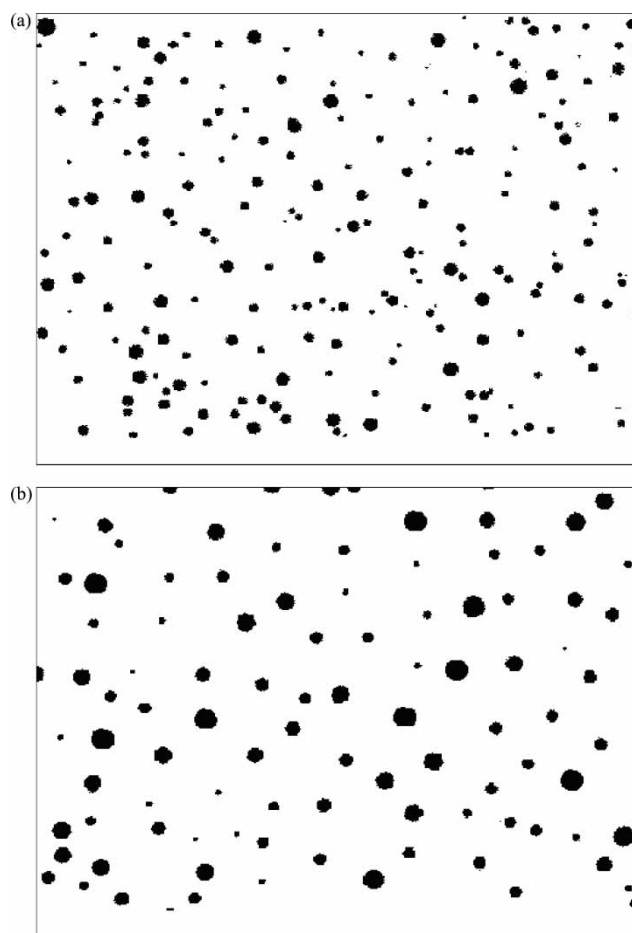


Fig. 4 Aerated oil sample: (a) 25 times magnification and (b) after standing for 10 min

bubble centre distance d_m ; these were the values taken to be effective at the bearing oil inlet. An aerator setting to maximize the number of air bubbles, and minimize the bubble size, resulted in $r_m = 0.0188 \text{ mm}$ and $d_m = 0.1114 \text{ mm}$, and this setting was used throughout the practical testing. Note that Fig. 4(b) is taken using the same sample as that in Fig. 4(a), but at 10 min later. It can be seen that given sufficient time the air bubbles tended to coalesce, resulting in larger but fewer air bubbles. Bearing testing and oil testing were therefore done simultaneously, so far as possible, so that an accurate record of the oil sample aeration characteristics could be noted; this was done by extracting oil samples from the oil supply line immediately next to the bearing oil inlet port, while bearing testing was taking place.

4 EXPERIMENTAL BEARING TESTING

Experimental testing was carried out using a rig (shown in Fig. 5) designed as a model turbogenerator bearing, with appropriate similarity in Sommerfeld number, and also designed to ensure that laminar oil flow was present in the bearing. The rig was designed to enable operation close to the limit of stability to be investigated as well as more normal operating conditions; experimental results were therefore collected for journal eccentricity ratios in the range 0.1 to 0.9. The rig consisted of a rigid rotor of mass 18.6 kg running in two identical plain journal bearings of diameter $D = 76.3 \text{ mm}$, length/diameter ratio $L/D = 1$, and bearing radial clearance $c = 0.5559 \text{ mm}$. The lubricant was supplied to the bearing at an inlet port on the horizontal centre line of the bearing, in the cavitated region of the bearing clearance; the oil exited the bearing in an axial direction and was collected at drain positions before being returned to the oil supply tank. The lubricating oil used for the work had a viscosity of $\eta = 0.148 \text{ Pa s}$, and tests were carried out at 16°C . Thermocouples were used to measure the temperature of the oil entering the bearing, and also that of the oil as it was collected following exit from the bearing; in all cases the temperature measured at each location was the same; it was, therefore, concluded that the viscosity of the oil did not change as it passed through the bearing. Movement of the test journal within the bearing clearance was measured using inductive displacement transducers located on the bearing housing at each end of the test bearing. The speed of the rotor was measured by means of a light source shining against a disc with radial slots mounted on the rotor; each time a slot passed the light source a sensor on the other side of the disc provided a signal to a data logging system.

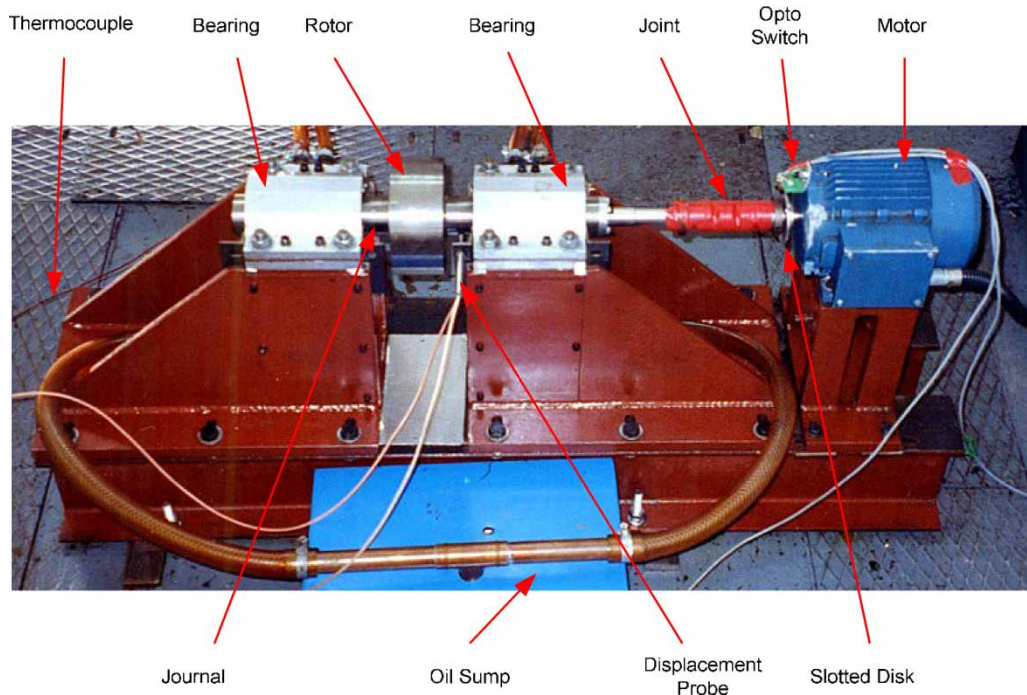


Fig. 5 Test rig used for the investigation

5 THEORETICAL AND EXPERIMENTAL RESULTS AND DISCUSSION

Theoretical results are presented in Figs 6 and 7 to show the effect of bubble size and of aeration level on bearing load. Figure 6 shows results for when bubble radius is one-fiftieth of the bearing radial clearance, and Fig. 7 for when it is one-hundredth of the bearing radial clearance. Of significance is that these figures show that the inclusion of air bubbles in the lubricating

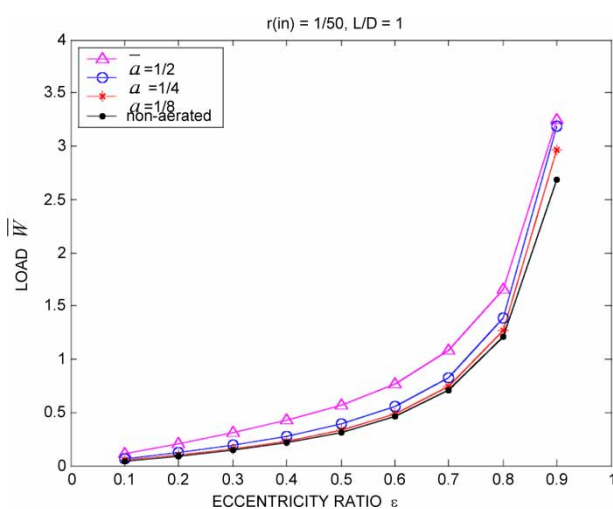


Fig. 6 Non-dimensional load versus eccentricity ratio for journal bearing lubricated by aerated oil with different aeration levels and $\bar{r} = 1/50$, $L/D = 1$, $D/2c = 1000$

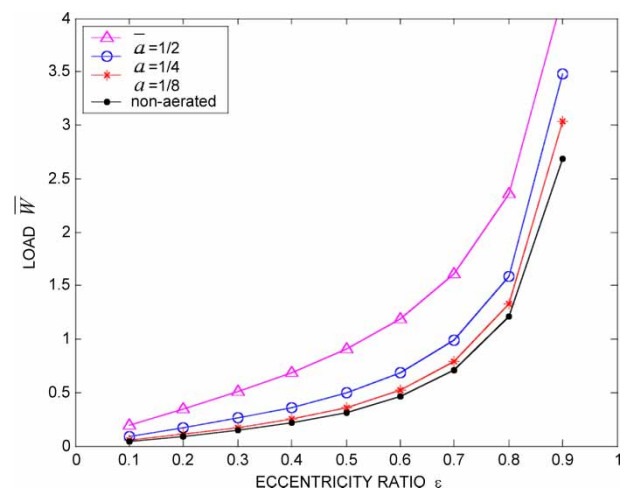


Fig. 7 Non-dimensional load versus eccentricity ratio for journal bearing lubricated by aerated oil with different aeration levels and $\bar{r} = 1/100$, $L/D = 1$, $D/2c = 1000$

oil tends to result in an increase in load-carrying capacity; this is contrary to the assumption that air bubbles result in a loss of load-carrying capacity. A possible explanation for this is that the surface tension properties of the air bubbles give rise to an effective increase in lubricant viscosity which more than offsets any decrease in load-carrying capacity caused by increased compressibility of the lubricant/air mix. It can also be seen that Fig. 7 shows potentially larger increases in load-carrying capacity than are shown in Fig. 6. This suggests that smaller bubbles have the

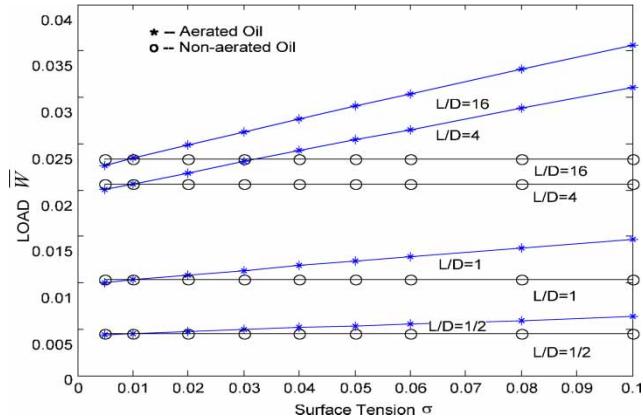


Fig. 8 Non-dimensional load versus surface tension at $\varepsilon = 0.7$ for non-aerated and aerated oil where $\bar{r} = 1/200$, $\bar{a} = 1/8$, $D/2c = 69$

potential to result in larger increases in load-carrying capacity. The other point of note from these two figures is that a closer spacing of the bubbles, for example when $\bar{a} = 0.5$, results in the greater increase in load-carrying capacity than when bubbles are more dispersed. Thus, in general, these results indicate that large numbers of small bubbles will have the greatest effect. (Further results for bubble radius equal to one-twentieth of the bearing radial clearance, not included for the sake of brevity, show only a very small increase in load-carrying capacity even for $\bar{a} = 0.5$.)

Figures 8 and 9 show further theoretical results, for aeration levels of $\bar{a} = 1/4$ and $1/8$, which show the influence of bubble surface tension on bearing load-carrying capacity. It can be seen that any increase in load-carrying capacity is proportional to surface tension. It is noteworthy also that for very low values of surface tension, less than 0.05 N/m, a reduction in load-carrying capacity is predicted.

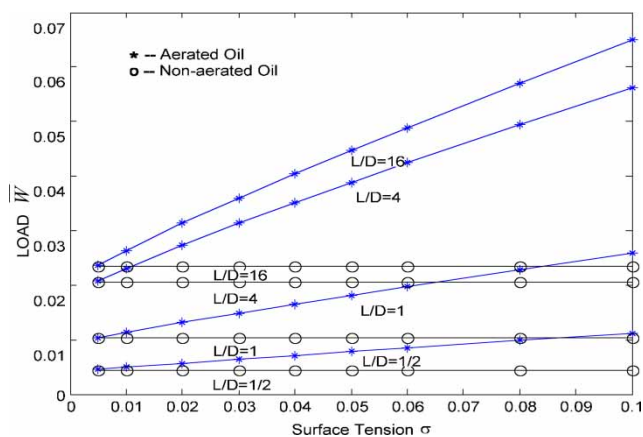


Fig. 9 Non-dimensional load versus surface tension at $\varepsilon = 0.7$ for non-aerated and aerated oil where $\bar{r} = 1/200$, $\bar{a} = 1/4$, $D/2c = 69$

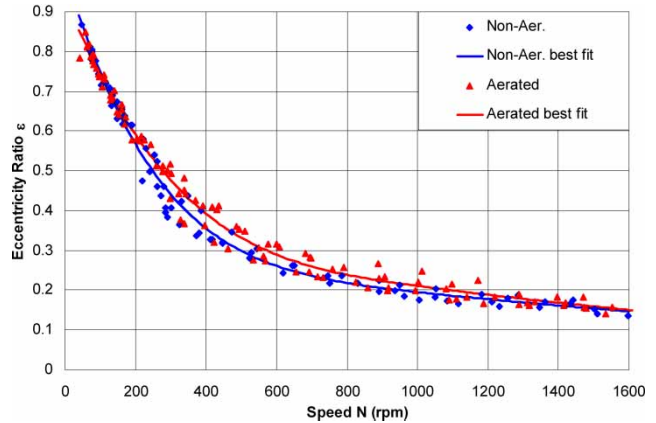


Fig. 10 Variation of bearing eccentricity ratio with rotor speed

It can be seen from the forgoing discussion that the greatest increase in load-carrying capacity is predicted by the theory to be brought about by having large numbers of small bubbles in a lubricant which has a high surface tension. The aerator setting giving the largest number of small bubbles was, therefore, used in the experimental stage of the investigation, as described earlier. This resulted in $\bar{a} = 1/6$ and $\bar{r} = 1/30$; the surface tension for the lubricant used was $\sigma = 0.0365$ N/m. The theoretical model was then re-run using these aeration conditions and the results superimposed on the experimental results.

Practical testing was carried out for both aerated and non-aerated oil at a range of operating speeds, and from the measurements values of bearing operating eccentricity ratio and attitude angle were computed. Figures 10 and 11 show how these quantities vary with rotor speed, and Fig. 12 shows how they relate to each other via the steady-state journal locus. Theoretical results are also shown in Fig. 12 for the purposes of comparison.

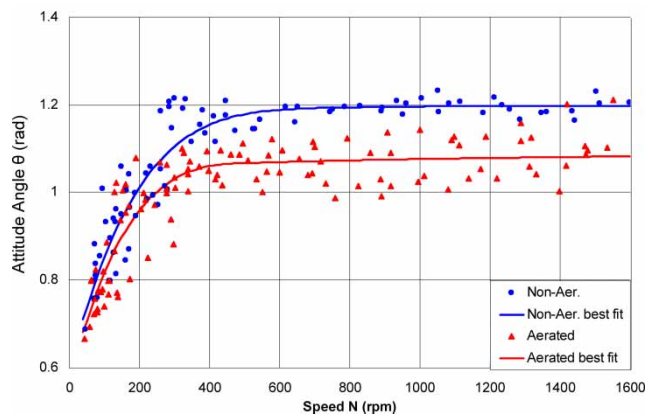


Fig. 11 Variation of bearing attitude angle with rotor speed

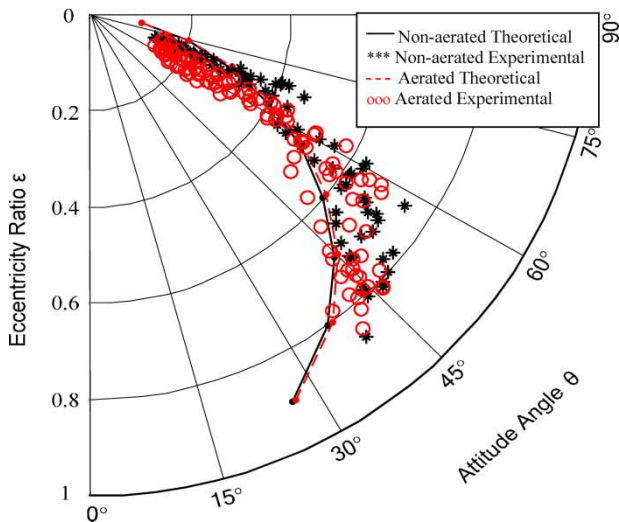


Fig. 12 Theoretical and experimental journal locus for aerated and non-aerated oil

Figure 10 shows the variation of eccentricity ratio with speed, for aerated and non-aerated oil. In general it can be seen that the effect of oil aeration is small throughout the whole of the speed range investigated. Of particular significance is the effect for eccentricity ratios greater than about 0.6 – the typical range for a journal bearing operating under load. It can be seen from the figure for results in this operating range in particular, the effect of oil aeration is negligible. The practical significance of eccentricity ratio is that a higher eccentricity ratio implies a greater risk of surface to surface contact at the bearing; the results in Fig. 10 show that, for eccentricity ratios above 0.6, this risk is not increased as a consequence of oil aeration, and so aeration appears not to affect the bearing load-carrying capacity adversely.

Figure 11 shows the variation of journal attitude angle with speed. It can be seen that in general oil aeration results in a smaller attitude angle, as compared with non-aerated oil, particularly so at higher operating speeds and attitude angles greater than about 1 rad. There is little practical significance in the attitude angle of the journal from a steady-load-carrying capacity perspective however.

Figure 12 shows the variation of eccentricity ratio with attitude angle. Again this figure shows that the aerated oil results in a smaller attitude angle, especially at higher operating speeds (when eccentricity ratio is less than 0.4 for example). Of particular interest however is the theoretical data, and its comparison with the experimental results. It can be seen that generally the theory predicts a steady-state journal locus that is barely affected by whether or not the lubricant contains air bubbles, and so this data too supports the argument that aeration of the lubricant has little effect on load-carrying capacity. Of note also is the

fact that the theoretical and experimental results show good agreement at low eccentricity ratios, although at eccentricity ratios greater than 0.6 the journal attitude angle tends to be slightly greater than predicted by theory.

Of note also is that while the theory presented above predicts that lubricant aeration can have the effect of increasing load-carrying capacity provided that there are very many very small closely spaced air bubbles, in practice this condition was found to be very difficult to achieve.

6 CONCLUSIONS

The work has shown that it is difficult to generate and sustain large numbers of small air bubbles within the lubricant, and that over a period of minutes smaller bubbles will tend to coalesce to form a smaller number of larger bubbles. An effective way of measuring oil aeration, however, is as described, using a photo-microscope and image analysis system.

In theory, provided that the air bubbles are small enough, and provided they are present in large numbers closely spaced, theory predicts that the presence of air bubbles in the lubricant can lead to an increase in load-carrying capacity because of the effect of surface tension, where surface tension is not too low. In practice, however, these conditions are difficult to achieve.

In practice, lubricant aeration results in a negligible effect on the load-carrying capacity of plain journal bearings. For any given load and speed, aeration will result either in only a slight increase in eccentricity ratio when the nominal eccentricity ratio is in any instance small initially, or no significant change at higher nominal eccentricity ratios more typical of normal operating conditions.

To summarize, the theoretical model (equations (1), (5), (10), (11), and (12)) for a journal bearing running on aerated oil shows that lubricant aeration has a negligible effect on the steady-load-carrying capacity for most practical conditions, and this is supported by the experimental observations.

REFERENCES

- 1 **Abdel-Latif, L. A., Peeken, H., and Benner, J.** Thermodynamic analysis of thrust bearing with circular pads running on bubbly oil (BTHD-theory). *ASME, J. Tribol.*, 1985, **107**(4), 527–537.
- 2 **Chamnprasart, K., Al-Sharif, A., Rajagopal, K. R., and Szeri, A. Z.** Lubrication with binary mixtures bubbly oil. *ASME, J. Tribol.*, 1993, **115**(2), 253–260.
- 3 **Nikolajsen, J. L.** The effect of aerated oil on the load capacity of a plain journal bearing. *ASME, Tribol. Trans.*, 1999, **42**(1), 58–62.

- 4 **Nikolajsen, J. L.** Viscosity and density models for aerated oil in fluid film bearings. *ASME, Tribol. Trans.*, 1999, **42**(1), 186–191.
- 5 **Calderwood, G. F. N., Douglas, H. W., and Mardles, E. W. J.** Anomalous flow of liquids through capillaries and measurement of viscosity. *Anal. Chem.*, 1953, **25**(6), 935–937.
- 6 **Hayward, A. T. J.** The viscosity of bubbly oil. Fluids report no. 99, National Engineering Laboratory, Glasgow, UK, 1961.
- 7 **Roach, M. P. and Goodwin, M. J.** Vibration control in rotating machinery by the use of accumulators or aerated lubricants. *Mach. Vibr.*, 1992, **1**, 211–215.
- 8 **An, Q., Yinsheng, Z., and Yongxin, Q.** Study on the viscosity properties of bubbly oil and the static characteristics of journal bearing lubricated with bubbly oil. *Wear*, 1997, **213**(1–2), 159–164.
- 9 **Diaz, S. E. and San Andres, L. A.** A method for identification of bearing force coefficients and its application to a squeeze film damper with a bubbly lubricant. *STLE Tribol. Trans.*, 1999, **42**, 739–746.
- 10 **Nikolajsen, J. L., Dong, D., and Goodwin, M. J.** Measurement of oil aeration effects in journal bearings. In ASME TurboExpo '02 Conference, Amsterdam, June 2002, ASME paper GT-2002-30290.
- 11 **Dong, D.** *The effect of oil aeration on the behaviour of journal bearings.* PhD Thesis, Staffordshire University, 2006.
- 12 **Smith, E. H.** The influence of surface tension on bearings lubricated with bubbly liquids. *ASME, J. Eng. Lubr. Technol.*, 1980, **103**, 91–96.

APPENDIX

Notation

\bar{a}	$=r/d$, aeration level (max $\bar{a} = 0.5$) (dimensionless)
A	surface area of bubble (m^2)
c	nominal radial clearance of the bearing (m)
d	distance between bubble centres (m)
d_m	mean bubble centre distance (m)
D	bearing diameter (m)
e	distance between bearing centre and journal centre (m)
h	bearing oil film thickness (m)
\bar{h}	$=h/c$ (dimensionless)
L	bearing length (m)
m	mass (kg)

p	pressure (N/m^2)
\bar{p}	$=\frac{p}{\rho_{\text{oil}}RT}$ (dimensionless)
r	bubble radius (m)
\bar{r}	$=r/c$, non-dimensional bubble radius (dimensionless)
r_m	mean bubble radius (m)
R	gas constant for air (Nm/kg K)
T	temperature (K)
U	journal surface velocity (m/s)
v	velocity of lubricant (m/s)
V	volume (m^3)
W	bearing load (N)
\bar{W}	$=2W/(DL\eta\omega[r/c]^2)$, dimensionless bearing load (dimensionless)
x	circumferential position in bearing (m)
y	radial position across lubricant film (m)
z	axial position in bearing (m)
\bar{z}	$=2z/L$ (dimensionless)
γ	ratio of specific heats for air (dimensionless)
Γ	dimensionless quantity $\frac{6\eta_{\text{oil}}\omega}{\rho_{\text{oil}}RT} \left(\frac{D}{2c}\right)^2$ (dimensionless)
ε	$=e/c$, eccentricity ratio (dimensionless)
η	dynamic viscosity (Pa s)
$\bar{\eta}$	$=\frac{\eta}{\eta_{\text{oil}}}$ (dimensionless)
μ	mass ratio $\frac{m_{\text{air}}}{m_{\text{oil}}}$ (dimensionless)
ρ	density (kg/m^3)
$\bar{\rho}$	$=\frac{\rho}{\rho_{\text{oil}}}$ (dimensionless)
σ	bubble surface tension (N/m)
φ	angular position in bearing oil film (rad)
ω	rotational speed (rad/s)
-	indicates dimensionless value

Subscripts

air	relates to the air
in	relates to condition at inlet to bearing
oil	relates to the oil
total	relates to net value for air and oil mixture