

# Backstepping-enhanced decentralised PID control for MIMO processes with an experimental study

Y. Zhang, S. Li and Q. Zhu

**Abstract:** A novel decentralised PID controller design procedure based on backstepping principles is presented to operate multiple-input multiple-output (MIMO) dynamic processes. First, a control Lyapunov function (CLF) and virtual control variable based on the backstepping method are derived recursively for each loop and a multivariable controller is obtained. Appropriate selection of auxiliary control variables can then eliminate coupling effects between subprocesses, and a decentralised controller can be derived. By analysing the regulation dynamics, external disturbances are rejected via appropriate tuning of the backstepping design parameters. Sufficient stability conditions are then derived using the small-gain theorem for two-input two-output and MIMO closed-loop systems, respectively. Simulation results for the control of the Shell benchmark problem and experimental results with a real-time water-level control system are provided to demonstrate the effectiveness and practicality of the proposed design procedure.

## 1 Introduction

Simply because of its relatively concise structure and fewer tuning parameters [1], PID control has been one of the most commonly used schemes for single-input single-output (SISO) processes in various industries, particularly chemical engineering. With their expanded control scope and greater accuracy multivariable PID control approaches that take loop interactions into consideration have increasingly received attention. However, the first difficulty in designing multivariable PID controllers is the higher dimensionality; for example, when the process model dimension is  $N$ , there will be  $3N^2$  tuning parameters for its multivariable PID controller, which is a large barrier for real-time control applications. The second difficulty is the loop coupling effect; that is, adjusting the controller parameters of one loop will affect the performance of the other loops, sometimes to the extent of destabilising the whole process. Accordingly, decentralised PID control [2] for MIMO processes has been a popular approach used in industry. Decentralised control has fewer tuning parameters ( $3N$  compared with the above centralized PID prototype), and is simple to design and implement. In addition, decentralised controller design focuses on a single control loop and has little effect on the other control loops, which is convenient for maintenance and amendment. However, the tuning procedure often involves trial-and-error experiments and requires experienced operators. Obviously this tuning style is not accurate, highly skill-related, and very time-consuming. It is the authors' belief that if the better structural features of decentralised control can be used, there will be a significant reduction in parameter tuning and

control system performance analysis. Some work has supported this belief. Halevi *et al.* [2] and Chen and Seborg [3] presented some effective decentralised PID control algorithms for complex interacting MIMO processes, in particular TITO cases. However, neither works gave guidance for stability analysis after the control systems had been designed.

Backstepping design is a recursive and systematic approach, originated by Kokotovic and co-workers in 1991 [4] and deriving from a nonlinear formulation. The design philosophy, for a class of feedback-linearisable nonlinear systems, is to decompose a complex system into multiple small-scale subsystems, then to design recursively a control Lyapunov function (CLF) [5] and virtual control variables for each subsystem, and finally to obtain the original control law for global regulation and tracking [6]. Some researchers have focused upon the application of backstepping methods to decentralised control. Zhang *et al.* [7] proposed an adaptive backstepping-based scheme for designing a totally decentralised adaptive stabiliser for a class of large-scale systems with guaranteed transient performance. It was shown that, with the proposed controller, global stability of the overall system and perfect regulation could be guaranteed, but the method is only suitable for large-scale systems that are not strongly interconnected. Unfortunately, for industrial PID control applications, the use of the backstepping approach is very limited [8–10]. Benaskeur and Desbiens [8] developed a backstepping-based adaptive PID control scheme whose robustness and transient performance are better than those of the conventional PID control. In [10], the same authors proposed a decentralised control scheme based on a modified control Lyapunov function for TITO linear plants. This showed that combining a backstepping scheme with decentralised PID control is of great theoretical and practiced value.

In this paper, a decentralised controller is derived by appropriate selection of the auxiliary control variable, which makes the tuning parameters of the PID controller equivalent to the designed parameters using a backstepping approach. It has been shown that the controller parameters obtained with the backstepping design for the first-order

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transfer function models are equivalent to a PI-type controller, and that for the first-order plus dead-time plant model, the backstepping design parameters are equivalent to a PID-type controller. By introducing the small-gain theorem, a sufficient condition for full closed-loop system stability is derived and the tracking performance and disturbances rejection of the system is improved.

## 2 Backstepping-enhanced multivariable control

In this section the decomposition mechanism for MIMO systems is explained, to form a basis for the parallel design of each subloop controller. CLF and virtual control variable based on backstepping are derived recursively for each loop to form an integrated process control strategy. It is shown that appropriate selection of auxiliary control variables can eliminate the coupling effect between subprocesses.

### 2.1 Configuration of decentralised control systems

A block diagram of a decentralised control for a MIMO process is shown in Fig. 1, where  $\mathbf{y}(t) = [y_1 \ y_2 \ \dots \ y_N]^T \in \mathbb{R}^N$  is the process output vector,  $\mathbf{u}(t) = [u_1 \ u_2 \ \dots \ u_N]^T \in \mathbb{R}^N$  is the process input (or controller output) vector,  $\mathbf{d}(t) = [d_1 \ d_2 \ \dots \ d_N]^T \in \mathbb{R}^N$  is the disturbance vector,  $\mathbf{y}_r(t) = [y_{r1} \ y_{r2} \ \dots \ y_{rN}]^T \in \mathbb{R}^N$  is the desired process reference signal (or setpoints) vector and  $\mathbf{e}(t) = [e_1 \ e_2 \ \dots \ e_N]^T \in \mathbb{R}^N$  is the loop error vector, the difference between the reference signal and output of the process,

$$\mathbf{G}_p = \begin{bmatrix} G_{11} & \dots & G_{1N} \\ \vdots & \ddots & \vdots \\ G_{N1} & \dots & G_{NN} \end{bmatrix}$$

is the process transfer function matrix.

For decentralised control, suppose that the input and output variables have been paired in terms of the principal diagonal of  $\mathbf{G}_p$ , that the control objectives of the decentralised control are to decompose the MIMO system into multiple control loops, to design multi-loop controllers, make the process outputs  $y_i(t)$  ( $i = 1, 2, \dots, N$ ) track the loop reference signals  $y_{ri}(t)$  respectively, and can make the full closed-loop control system asymptotically stable, and that the process has desired property for disturbance rejection. In decentralised control, the controller  $\mathbf{G}_c$  is a diagonal matrix given by  $\mathbf{G}_c = \text{diag}\{G_{c1}, G_{c2}, \dots, G_{cN}\}$ , where  $G_{ci}$  ( $i = 1, 2, \dots, N$ ) is considered as a PID controller.

With regard to the decentralized control, the control system shown in Fig. 1 is decomposed into  $N$  control loops and the controllers for the loops are designed simultaneously. For the  $i$ th control loop,  $y_{ii}$  is the output response to the input  $u_i$ , the control effort on the  $i$ th sub process from other control inputs is denoted by  $y_{ci}$ , and  $Y_{ci}(s) = \sum_{j=1, j \neq i}^N G_{ij}(s)U_j(s)$ , where  $U_i(s)$  and  $Y_{ci}(s)$  are the Laplace transforms of  $u_i$  and  $y_{ci}$ ,  $y_i$  is the output response

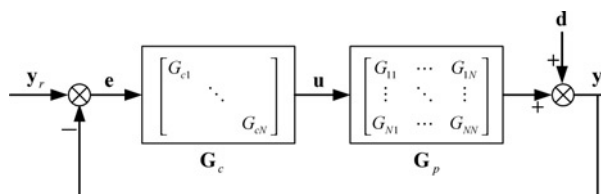


Fig. 1 MIMO decentralised control system

of the  $i$ th control loop, and  $Y_i(s) = Y_{ii}(s) + Y_{ci}(s)$ , where  $Y_i(s)$  and  $Y_{ii}(s)$  are the Laplace transforms of  $y_i$  and  $y_{ii}$ .

### 2.2 Multivariable control with backstepping principle

For the  $i$ th control loop,  $Y_{ri}(s)$  and  $E_i(s)$  are the Laplace transforms of  $y_{ri}$  and  $e_i$ , respectively. The minimum-phase transfer function of the  $i$ th sub process is expressed as

$$G_{ii}(s) = \frac{Y_{ii}(s)}{U_i(s)} = \frac{B_i(s)}{A_i(s)} = \frac{s^{m_i} + b_{i,m_i-1}s^{m_i-1} + \dots + b_{i0}}{s^{n_i} + a_{i,n_i-1}s^{n_i-1} + \dots + a_{i0}} \quad (m_i < n_i) \quad (1)$$

Here the decentralised PID control for MIMO processes is developed on the basis of the original work of Benaskeur and Desbiens [8, 10]. Compared with [10], an integral action is inserted into the control input,  $\bar{U}_i(s) = sB_i(s)U_i(s)$ ,  $\bar{u}_i$  is seen as a new control input and  $\bar{U}_i(s)$  is the Laplace transform of  $\bar{u}_i$ . Then the transfer function of the  $i$ th sub process can be rewritten as

$$\bar{G}_{ii}(s) = \frac{Y_{ii}(s)}{\bar{U}_i(s)} = \frac{1}{sA_i(s)} = \frac{1}{s^{n_i+1} + a_{i,n_i-1}s^{n_i} + \dots + a_{i0}s} \quad (2)$$

and its state-space representation can be expressed as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n_i} &= x_{n_i+1} \\ \dot{x}_{n_i+1} &= -a_{i0}x_2 - \dots - a_{i,n_i-1}x_{n_i+1} + \bar{u}_i \\ y_{ii} &= x_1 \end{aligned} \quad (3)$$

It has been shown that the backstepping method is applicable for these lower-triangle nonlinear or accurate linearised systems [4]. The state-space equation (3) is a special lower-triangle linear model representation; therefore the backstepping method can be used recursively to design controllers for each loop. The design procedure for the  $i$ th control loop can be described as follows.

*Step 1:* The first error variable is defined as

$$\begin{aligned} z_1 &= y_i - y_{ri} + \varepsilon_i = y_{ii} + y_{ci} - y_{ri} + \varepsilon_i \\ &= x_1 + y_{ci} - y_{ri} + \varepsilon_i \end{aligned} \quad (4)$$

where  $\varepsilon_i$  [10] is an auxiliary control variable used in the decentralised controller design in Section 3.1. Choosing the first CLF,  $V_1 = (1/2)z_1^2$ , its derivative is

$$\dot{V}_1 = z_1(x_2 + \dot{y}_{ci} - \dot{y}_{ri} + \dot{\varepsilon}_i) \quad (5)$$

The term  $x_2 + \dot{y}_{ci}$  is taken as the first virtual control variable, with desired value

$$\alpha_1 = (x_2 + \dot{y}_{ci})_d = -c_1 z_1 + \dot{y}_{ri} - \dot{\varepsilon}_i \quad (6)$$

where  $c_1 > 0$  is the backstepping design parameter. With the above choice, (5) becomes negative-definite.



1. For the first-order model, that is,  $n_i = 1, m_i = 0$ ,  $\mathbf{C}_2 = \begin{bmatrix} -c_1 & 1 \\ -1 & -c_2 \end{bmatrix}$ ,  $Q_i(s) = |s\mathbf{I}_2 - \mathbf{C}_2| = s^2 + (c_1 + c_2)s + c_1c_2 + 1$ , and the control input of the  $i$ th loop is

$$\begin{aligned} U_i(s) &= \frac{(c_1 + c_2 - a_{i0}) + (c_1c_2 + 1)/s}{b_{i0}} E_i(s) + G_{ii}^{-1}(s) Y_{ri}(s) \\ &= (K_P + K_I/s) E_i(s) + G_{ii}^{-1}(s) Y_{ri}(s) \\ &= G_{PI}(s) E_i(s) + G_{ii}^{-1}(s) Y_{ri}(s) \end{aligned} \quad (18)$$

where PI-type controller parameters are obtained from

$$\begin{aligned} K_P &= (c_1 + c_2 - a_{i0})/b_{i0} \\ K_I &= (c_1c_2 + 1)/b_{i0} \end{aligned} \quad (19)$$

In the presence of model uncertainties and/or disturbances, the PI-type controller makes appropriate corrections by tuning  $c_i$ , which can be treated as a pole-placement scheme. The regulation dynamics is

$$\frac{Y_i(s)}{D_i(s)} = \frac{sA_i(s)}{Q_i(s)} \quad (20)$$

where  $D_i(s)$  is the Laplace transform of  $d_i$ . The regulation poles depend on  $Q_i(s)$ , and, for simplicity, two poles are placed at the same real value  $p_i$  ( $p_i < 0$ ); that is,  $Q_i(s) = (s - p_i)^2$ . Then  $s^2 + (c_1 + c_2)s + c_1c_2 + 1 = (s - p_i)^2$  and the possible values for the design parameters are selected with

$$\begin{aligned} c_1 &= -p_i + 1 \\ c_2 &= -p_i - 1 \end{aligned} \quad (21)$$

with  $p_i < -1$  to ensure the positivity of  $c_2$ .

2. For the FOPDT model,

$$G_{ii}(s) = \frac{K_i}{T_i s + 1} e^{-\tau_i s}$$

where  $K_i$  is the steady-state gain,  $T_i$  is the time constant, and  $\tau_i$  is the time delay, and the timescale must be changed by dividing by  $\beta$  (the ratio of the timescale transform,  $\beta \gg \tau_j, j = 1, 2, \dots, N$ ) and the new transfer function used to design for the subcontroller can be described as a second-order model given by

$$G_{ii}^c(s) = \frac{b_{i0}}{s^2 + a_{i1}s + a_{i0}}$$

where

$$a_{i1} = \frac{(T_i + \tau_i)\beta}{T_i\tau_i}, \quad a_{i0} = \frac{\beta^2}{T_i\tau_i}, \quad b_{i0} = \frac{K_i\beta^2}{T_i\tau_i}$$

with the new time scale. According to the above design procedure,

$$\mathbf{C}_3 = \begin{bmatrix} -c_1 & 1 & 0 \\ -1 & -c_2 & 1 \\ 0 & -1 & -c_3 \end{bmatrix},$$

$$\begin{aligned} Q_i(s) &= |s\mathbf{I}_3 - \mathbf{C}_3| \\ &= s^3 + (c_1 + c_2 + c_3)s^2 + (c_1c_2 + c_2c_3 + c_1c_3 + 2)s \\ &\quad + c_1c_2c_3 + c_1 + c_3 \end{aligned}$$

and the control input of the  $i$ th loop is

$$\begin{aligned} U_i(s) &= \frac{(c_1c_2 + c_2c_3 + c_1c_3 + 2 - a_{i0}) + (c_1c_2c_3 + c_1 + c_3)/s}{b_{i0}} E_i(s) + G_{ii}^c(s)^{-1} Y_{ri}(s) \\ &= (K_P + K_I/s + K_D s) E_i(s) + G_{ii}^c(s)^{-1} Y_{ri}(s) \\ &= G_{PID}(s) E_i(s) + G_{ii}^c(s)^{-1} Y_{ri}(s) \end{aligned} \quad (22)$$

where the PID-type controller parameters are obtained from

$$\begin{aligned} K_P &= (c_1c_2 + c_2c_3 + c_1c_3 + 2 - a_{i0})/b_{i0} \\ K_I &= (c_1c_2c_3 + c_1 + c_3)/b_{i0} \\ K_D &= (c_1 + c_2 + c_3 - a_{i1})/b_{i0} \end{aligned} \quad (23)$$

Similarly to the above procedure, three poles are placed at the same real value  $p_i$  ( $p_i < 0$ ), and the possible values for the design parameters are selected as

$$\begin{aligned} c_1 &= -p_i + \sqrt{2} \\ c_2 &= -p_i \\ c_3 &= -p_i - \sqrt{2} \end{aligned} \quad (24)$$

where  $p_i < -\sqrt{2}$  to ensure the positivity of  $c_3$ .

*Remark 1:* It has been noticed that the conventional PID controller parameters  $K_P$ ,  $K_I$ , and  $K_D$  are linked with backstepping-based design parameters  $c_i$  ( $> 0$ ). For the first-order process model, PI-type controllers are feasible, and equivalent to those designed via the backstepping approach. For the FOPDT process model, PID-type controllers are feasible and equivalent to those designed via backstepping approach. For higher-order process models, the proposed backstepping method leads to higher-order PID-type controllers. An appropriate selection of  $c_i$  would make a process well equipped to reject disturbances. In addition, PID controller parameters are dependent on the process model dynamics/parameters, and adaptive PI/PID-type decentralised controllers could be designed to accommodate process dynamic uncertainties. It should be noted that although the design procedure for PID controllers has not been directly adapted to deal with hard constraints, it is hoped that this will be seriously considered in future studies. Tentatively is the proposed design approach will be applied to a simulation example with hard constraints in this study to obtain some useful experience.

### 3.2 Closed-loop stability analysis

To guarantee the nominal stability of the closed-loop system, the following assumptions are made for the process models [10]:

1.  $B_i(s)$  ( $i = 1, 2, \dots, N$ ) is a Hurwitz polynomial.
2. The interaction transfer function  $G_{ij}(s)$  ( $i, j = 1, 2, \dots, N, i \neq j$ ) is strictly stable.

For the  $i$ th control loop,  $n_i + 1$  error variables are defined as

$$\begin{aligned} z_1 &= y_i - y_{ri} + \varepsilon_i \\ z_l &= x_l + y_{ci}^{(l-1)} - \alpha_{l-1}, \quad l = 2, \dots, n_i + 1 \end{aligned} \quad (25)$$

Accordingly, the following  $n_i + 1$  differential equations are derived

$$\begin{aligned} \dot{z}_1 &= -c_1 z_1 + z_2 \\ \dot{z}_l &= -z_{l-1} - c_l z_l + z_{l+1}, l = 2, \dots, n_i \\ \dot{z}_{n_i+1} &= -z_{n_i} - c_{n_i+1} z_{n_i+1} \end{aligned} \quad (26)$$

A positive-definite CLF is chosen as

$$V_{n_i+1} = \frac{1}{2} \sum_{l=1}^{n_i+1} z_l^2 > 0 \quad (27)$$

and its derivative is consequently derived as

$$\begin{aligned} \dot{V}_{n_i+1} &= \sum_{l=1}^{n_i+1} z_l \dot{z}_l = z_1(-c_1 z_1 + z_2) \\ &+ \sum_{l=2}^{n_i} z_l(-z_{l-1} - c_l z_l + z_{l+1}) \\ &+ z_{n_i+1}(-z_{n_i} - c_{n_i+1} z_{n_i+1}) = -\sum_{l=1}^{n_i+1} c_l z_l^2 < 0 \end{aligned} \quad (28)$$

which is negative-definite. Then  $V_{n_i+1}$  is bounded,  $z_l(l = 1, \dots, n_i + 1)$  is bounded [11], and

$$\lim_{t \rightarrow \infty} z_l(t) = 0, \quad l = 1, \dots, n_i + 1 \quad (29)$$

In addition, for the  $i$ th control loop, selecting

$$F_i(s) = -\frac{sA_i(s)}{Q_i(s)} Y_{ci}(s)$$

if  $y_{ci}(t)$  is bounded (this will be proved below), then the following expression holds according to the final-value theorem

$$\begin{aligned} \lim_{t \rightarrow \infty} \varepsilon_i(t) &= \lim_{s \rightarrow 0} sF_i(s) \\ &= -\lim_{s \rightarrow 0} \frac{s^{n_i+2} + a_{i,n_i-1}s^{n_i+1} + \dots + a_{i0}s^2}{s^{n_i+1} + q_{i,n_i}s^{n_i} + \dots + q_{i1}s + q_{i0}} Y_{ci}(s) = 0 \end{aligned} \quad (30)$$

From (25), (29) and (30),

$$\begin{aligned} \lim_{t \rightarrow \infty} z_1(t) &= \lim_{t \rightarrow \infty} [y_i(t) - y_{ri}(t) + \varepsilon_i(t)] \\ &= \lim_{t \rightarrow \infty} [y_i(t) - y_{ri}(t)] = 0 \end{aligned} \quad (31)$$

Hence the process output  $y_i(t)$  tracks asymptotically the loop reference signal  $y_{ri}(t)$ , the  $i$ th closed-loop subsystem is asymptotically stable. The controller  $\mathbf{G}_c = \text{diag}\{G_{c1}, G_{c2}, \dots, G_{cN}\}$  proposed in Section 3.1 is to be designed for the decentralized process  $\bar{\mathbf{G}}_p = \text{diag}\{G_{11}, G_{22}, \dots, G_{NN}\}$  such that the diagonal closed-loop system with transfer matrix  $\bar{\mathbf{G}}_h = \text{diag}\{\bar{G}_{h1}, \bar{G}_{h2}, \dots, \bar{G}_{hN}\}$  is stable, where

$$\bar{G}_{hi} = \frac{G_{ii}G_{ci}}{1 + G_{ii}G_{ci}}$$

It should be noted that the small-gain theorem [12] can be used to obtain a sufficient condition for determining the stability of the whole closed-loop system  $\mathbf{G}_h = \mathbf{G}_p \mathbf{G}_c (\mathbf{I} + \mathbf{G}_p \mathbf{G}_c)^{-1}$ , in which the closed-loop system is stable if the spectral radius (the maximum eigenvalue of a matrix)

or the infinite norm

$$\begin{aligned} \rho(\mathbf{E}_p(j\omega)\bar{\mathbf{G}}_h(j\omega)) &< 1 \quad \text{or} \\ \|\mathbf{E}_p(j\omega)\bar{\mathbf{G}}_h(j\omega)\|_\infty &< 1 \quad \forall \omega \end{aligned} \quad (32)$$

where  $\mathbf{E}_p = (\mathbf{G}_p - \bar{\mathbf{G}}_p)\bar{\mathbf{G}}_p^{-1}$  represents the multiplicative error between the full MIMO and decentralised models. In particular, for TITO processes,

$$\begin{aligned} \mathbf{E}_p &= \begin{bmatrix} 0 & \frac{G_{12}}{G_{22}} \\ \frac{G_{21}}{G_{11}} & 0 \end{bmatrix}, \bar{G}_{hi} = 1 - \frac{sA_i}{Q_i} \quad (i = 1, 2) \\ \mathbf{E}_p \bar{\mathbf{G}}_h &= \begin{bmatrix} 0 & \frac{G_{12}}{G_{22}} \left(1 - \frac{sA_2}{Q_2}\right) \\ \frac{G_{21}}{G_{11}} \left(1 - \frac{sA_1}{Q_1}\right) & 0 \end{bmatrix} \end{aligned}$$

and then a sufficient condition for the stability of the whole closed-loop system is

$$\left\| \frac{G_{12}G_{21}}{G_{11}G_{22}} \left(1 - \frac{sA_1}{Q_1}\right) \left(1 - \frac{sA_2}{Q_2}\right) \right\|_\infty < 1 \quad (33)$$

For  $N$ -input  $N$ -output processes ( $N \geq 3$ ),

$$\mathbf{E}_p = \begin{bmatrix} 0 & G_{12}G_{22}^{-1} \\ G_{21}G_{11}^{-1} & 0 \\ \vdots & \ddots \\ G_{N1}G_{11}^{-1} & \dots \\ \dots & G_{1N}G_{NN}^{-1} \\ \vdots & \vdots \\ \vdots & G_{N-1,N}G_{NN}^{-1} \\ G_{N,N-1}G_{N-1,N-1}^{-1} & 0 \end{bmatrix}$$

and then a sufficient condition for the stability of the whole closed-loop system is

$$\left\| \begin{bmatrix} 0 & G_{12}G_{22}^{-1}\bar{G}_{h2} \\ G_{21}G_{11}^{-1}\bar{G}_{h1} & 0 \\ \vdots & \ddots \\ G_{N1}G_{11}^{-1}\bar{G}_{h1} & \dots \\ \dots & G_{1N}G_{NN}^{-1}\bar{G}_{hN} \\ \vdots & \vdots \\ \vdots & G_{N-1,N}G_{NN}^{-1}\bar{G}_{hN} \\ G_{N,N-1}G_{N-1,N-1}^{-1}\bar{G}_{hN-1} & 0 \end{bmatrix} \right\|_\infty < 1 \quad (34)$$

Therefore, when the condition (34) is satisfied, MIMO processes can be stabilised using multiple decentralised controllers.

## 4 Simulation and experiment

### 4.1 Simulation for Shell benchmark control problem

A typical multivariable control problem has been considered as a case study to demonstrate the developed

design procedure, namely the control of a Shell heavy oil fractionator with three product draws and three side circulating loops [13]. The key elements of the Shell benchmark control problem are shown in Fig. 3, and the process input/output relations are modelled linearly using a matrix of FOPDT transfer functions.

Prett and Morari [13] presented a model for heavy oil fractionators for the Shell standard control problem

$$\mathbf{y} = \mathbf{G}_p(s)\mathbf{u} + \mathbf{G}_d(s)\mathbf{d} \quad (35)$$

where  $\mathbf{u} = [u_1 \ u_2 \ u_3]^T$  are manipulated variables to control the process,  $u_1$  and  $u_2$  represent the product draw rates from top and side of the column, respectively, and  $u_3$  representing the reflux heat duty for the bottom of the column;  $\mathbf{d} = [d_1 \ d_2]^T$  are unmeasured but bounded disturbances entering the column,  $d_1$  representing the reflux heat duty for the intermediate section of the column and  $d_2$  the reflux heat duty for the top of the column, with  $|d_1| \leq 0.5$  and  $|d_2| \leq 0.5$ ;  $\mathbf{y} = [y_1 \ y_2 \ y_3]^T$  are output variables, with  $y_1$  and  $y_2$  for the draw composition from the top and the side of the column, respectively, and  $y_3$  for the reflux temperature at the bottom of the column;  $\mathbf{G}_p(s)$  and  $\mathbf{G}_d(s)$  are the process and disturbance transfer-function matrices. The parameters of the nominal model are shown in Table 1 (the units for  $T$  and  $\tau$  are minutes).

The main control objective of the whole system is to maintain the draw composition from the top ( $y_1$ ) and the side ( $y_2$ ) of the column within a specified interval ( $0.0 \pm 0.005$  at steady state). The constraints on the output variables and the manipulated variables are set with  $|y_i| \leq 0.5 (i = 1, 2), y_3 \geq -0.5$  and  $|u_i| \leq 0.5, |\Delta u_i| \leq 0.05 (i = 1, 2, 3)$ , respectively.

It can be seen that the Shell benchmark control is an extremely complex problem with many, possibly conflicting, process requirements that are very difficult to satisfy simultaneously. A representative Shell benchmark control approach is the quadratic dynamic matrix control (QDMC) algorithm [14]. The main advantage of QDMC is that the objectives and constraints associated with the control problem are embedded in the control algorithm, which requires only minimal *ad hoc* controller adjustments. However, an obvious drawback of QDMC is that it uses a centralised control scheme, leading to an enormous online computational burden and even potential dangers in process operation. Thus it has become very important to develop computationally efficient control architectures and algorithms with simple implementative, such as the decentralised control and PID control schemes that are widely accepted in most industries.

With regard to decentralised control, first of all, loop pairing is derived using relative gain array (RGA) [15] and the Niederlinski index (NI) rules [16]. The steady-state

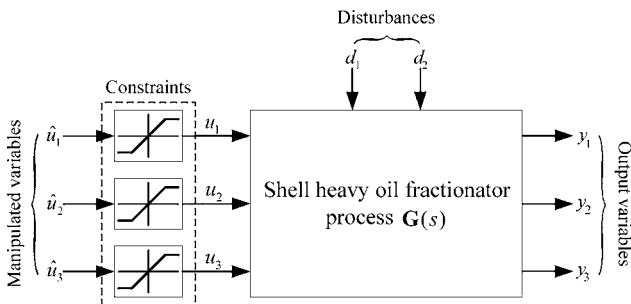


Fig. 3 Shell benchmark control problem

Table 1: Shell heavy oil fractionator nominal model parameters  $(K/Ts + 1e)^{-\tau s}$

	$u_1$	$u_2$	$u_3$	$d_1$	$d_2$
$y_1$	$K = 4.05$ $T = 50$ $\tau = 27$	$K = 1.77$ $T = 60$ $\tau = 28$	$K = 5.88$ $T = 50$ $\tau = 27$	$K = 1.20$ $T = 45$ $\tau = 27$	$K = 1.44$ $T = 40$ $\tau = 27$
$y_2$	$K = 5.39$ $T = 50$ $\tau = 18$	$K = 5.72$ $T = 60$ $\tau = 14$	$K = 6.90$ $T = 40$ $\tau = 15$	$K = 1.52$ $T = 25$ $\tau = 15$	$K = 1.83$ $T = 20$ $\tau = 15$
$y_3$	$K = 4.38$ $T = 33$ $\tau = 20$	$K = 4.42$ $T = 44$ $\tau = 22$	$K = 7.20$ $T = 19$ $\tau = 0$	$K = 1.14$ $T = 27$ $\tau = 0$	$K = 1.26$ $T = 32$ $\tau = 0$

gain matrix of  $\mathbf{G}_p(s)$  is

$$\mathbf{G}_p(0) = \begin{bmatrix} 4.05 & 1.77 & 5.88 \\ 5.39 & 5.72 & 6.90 \\ 4.38 & 4.42 & 7.20 \end{bmatrix}$$

The steady-state RGA of  $\mathbf{G}_p(0)$  is

$$\mathbf{\Lambda} = \begin{bmatrix} 2.0757 & -0.7289 & -0.3468 \\ 3.4242 & 0.9343 & -3.3585 \\ -4.4999 & 0.7946 & 4.7053 \end{bmatrix}$$

It can be seen that  $\mathbf{\Lambda}$  has positive elements on the main diagonal, and by the RGA rule, the pairing of input and output variables is determined as  $(u_1, y_1)$ ,  $(u_2, y_2)$ , and  $(u_3, y_3)$ . Then the NI, used in conjunction with the RGA-based rule for loop pairing, is

$$NI = 0.1250 > 0$$

With the above pairing, the Shell control system is structurally stable.

A sampling time of 4 minutes and a simulation time of 400 minutes are considered. Because the elements of  $\mathbf{G}_p(s)$  are the FOPDT transfer functions with large time delay, the timescale must be changed, and the ratio of the timescale transform is selected as  $\beta = 100$ . Then the new transfer functions used to design for subcontroller are

$$G_{11}^c(s) = \frac{4.05}{(0.5s + 1)(0.27s + 1)},$$

$$G_{22}^c(s) = \frac{5.72}{(0.6s + 1)(0.14s + 1)}, \quad G_{33}^c(s) = \frac{7.20}{0.19s + 1}$$

The backstepping design parameters for each control loop are specified as

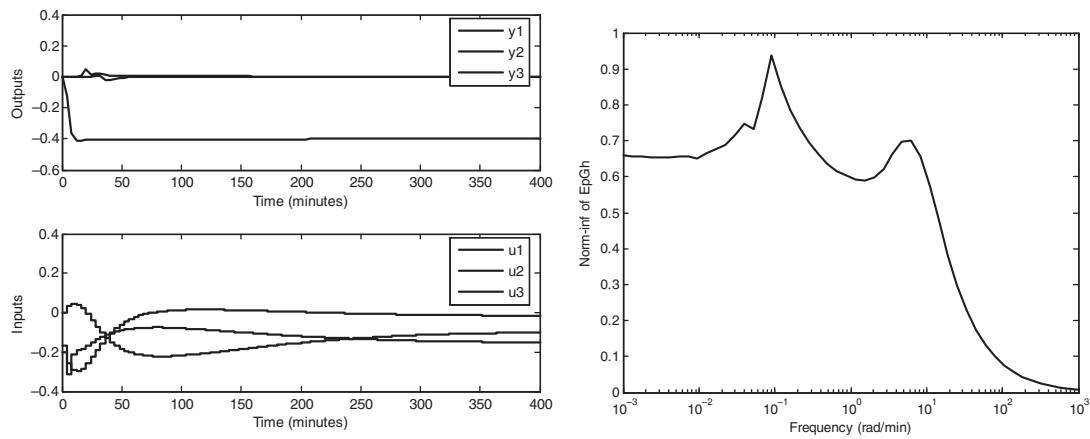
$$c_1^1 = 4.0809, \quad c_2^1 = 2.6667, \quad c_3^1 = 1.2525$$

$$c_1^2 = 6.2627, \quad c_2^2 = 4.8485, \quad c_3^2 = 3.4343 \quad (36)$$

$$c_1^3 = 3, \quad c_2^3 = 1$$

A new sampling time is set as 0.04 minutes. Three PID controllers are independently designed using the proposed backstepping method for the above three sub processes. In order to test the performance of the control scheme, the closed-loop system is subjected to disturbance patterns  $\mathbf{d}^1 = [0.5 \ 0.5]^T$  and  $\mathbf{d}^2 = [-0.5 \ -0.5]^T$ . This means that  $\mathbf{d}^1$  and  $\mathbf{d}^2$  represent the worst-case scenarios, since  $d_1$  and  $d_2$  are at the extremes of  $\pm 0.5$  and have the same sign.

The Matlab based simulation results (Figs. 4 and 5) show the process output responses, the control input



**Fig. 4** Simulation results for the disturbance pattern  $d^1$

a System output responses and control input signals  
 b  $\|E_p \bar{G}_h\|_\infty$

signal variations, and the corresponding sufficient conditions for the stability of the whole closed-loop system  $\|E_p \bar{G}_h\|_\infty$  [17], under the disturbance patterns  $d^1$  and  $d^2$ , respectively.

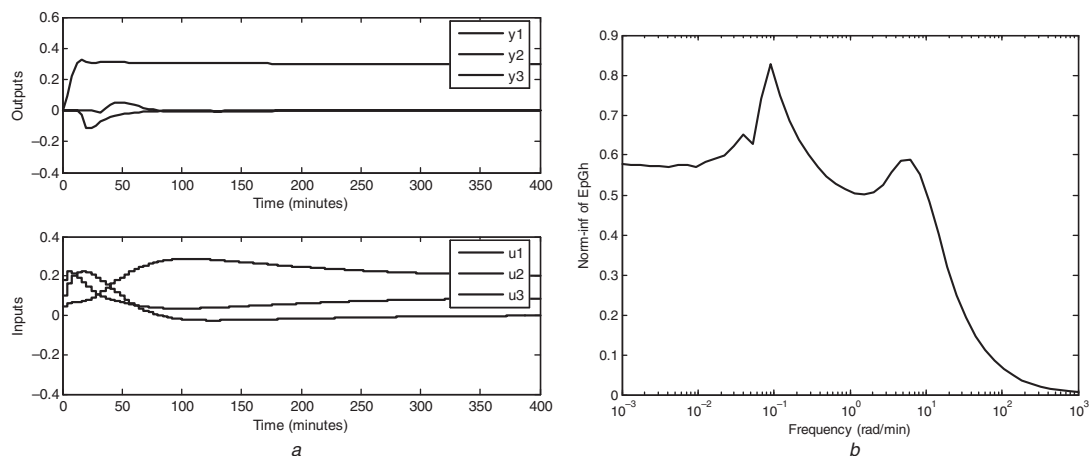
It can be seen from Figs 4 and 5 that, under two disturbance test patterns, the process output responses are stable and all control input signals are within their amplitude and rate limit constraints. Since the decentralised controllers have  $\|E_p \bar{G}_h\|_\infty < 1$  for all frequencies, the stability of the whole closed-loop system can be guaranteed. In addition, the design parameters for each loop under decentralised control can be designed and tuned separately. This can be simple to implement, in contrast to the more advanced but computationally intensive QDMC approach. It should be noted that although the proposed design procedure is not directly linked with control of hard constraints, it works well, at least for this case study. A thorough theoretical study will be devoted to the expansion of the procedure to the control of systems with hard constraints.

#### 4.2 Experiment on water-level control system

An experiment was carried out to evaluate the proposed decentralised PID controller design formula through performance comparisons with conventional PID controllers.

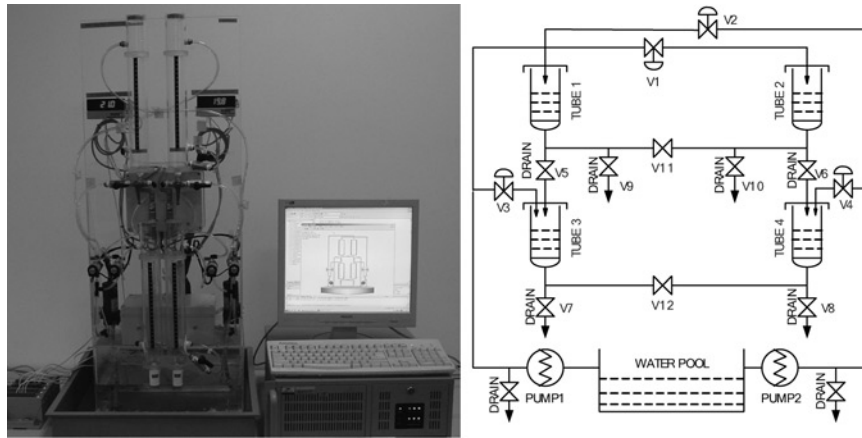
The plant employed in the experiment is a coupled-tube tank system (shown in Fig. 6), consisting of four Plexiglas tubes (TUBE1-TUBE4) divided into up and down tubes by a center partition. Water is pumped from a reservoir into the tubes by two variable-speed pumps (PUMP1 and PUMP2), driven by two electrical motors. The flow rate is controlled by an on/off valve, consisting of a cylindrical bob weight inside a cylindrical tube. The controlled variables in the experimental system are the water levels in TUBE3 and TUBE4, the inlet flow rates control variables, and the valves V3 and V4 are manipulated variables. An interaction valve V12 couples TUBE3 and TUBE4, and it can change the extent of coupling between TUBE3 and TUBE4. By closing interaction valve V12, the control system can be seen as two SISO subprocesses. The inlet flow rates from the up tubes (TUBE1 and TUBE2) to the down tubes (TUBE3 and TUBE4) are considered as disturbances by the regulating valves V5 and V6. Therefore, the water level control system can be considered as a nonlinear TITO process with strong coupling and various disturbances.

The water level control system can be approximated by a first- or second-order plus dead-time transfer function matrix model for various operating conditions. The parameter identification method proposed in [18] was used



**Fig. 5** Simulation results for the disturbance pattern  $d^2$

a System output responses and control input signals  
 b  $\|E_p \bar{G}_h\|_\infty$



**Fig. 6** Schematic diagram of a coupled-tube tanks apparatus

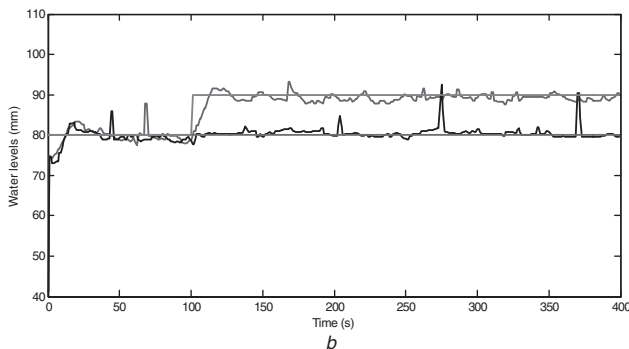
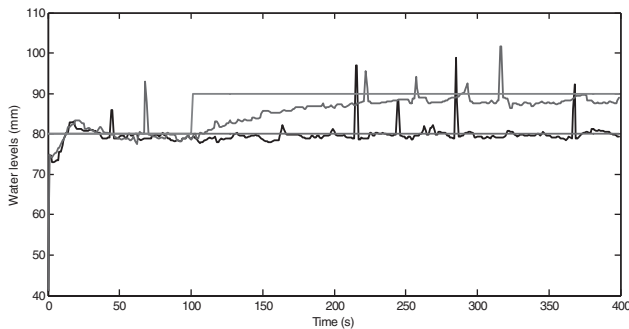
for this experimental plant. Through sequential step change of setpoints, the coupled closed-loop TITO system is decoupled equivalently into four independent single open-loop processes with the same input signal acting on the four transfer functions. Consequently, the parameters of first- or second-order plus dead-time models for each transfer function can be obtained directly using the linear regression equations derived for the decoupled identification system.

A sampling time of 0.5 s was considered. Two test cases were provided in this experiment: Case 1 was employed to compare regulating and tracking performances between the proposed PID-type controller design formula (called the backstepping-PID strategy) and conventional PID controllers with fixed parameters; case 2 was employed to compare the performance of disturbance rejection between the backstepping-PID strategy and conventional PID controllers.

*Case 1:* At the beginning, the setpoints of TUBE3 and TUBE4 were all kept at 80 mm, and the coupled valve

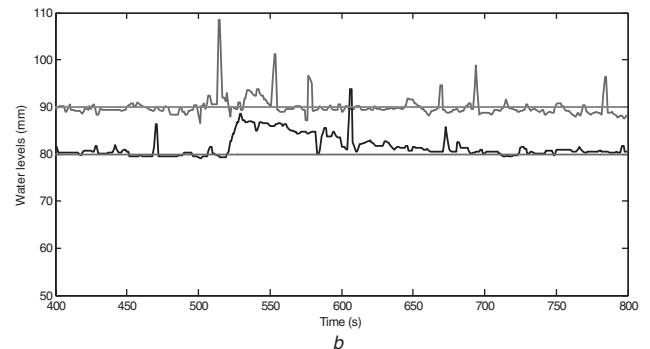
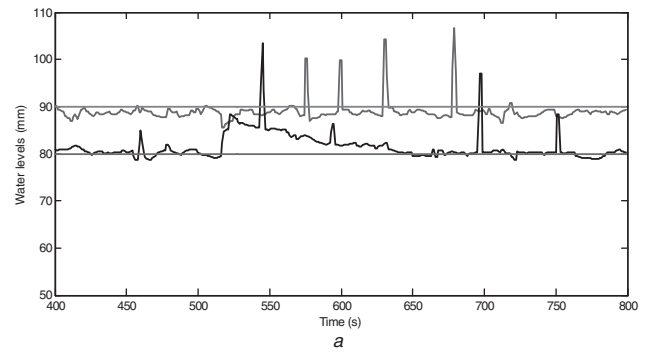
was closed. Optimal conventional PID parameters, obtained by modified Ziegler-Nichols tuning rules, were selected with  $K_P = 2$ ,  $K_I = 10$  and  $K_D = 1$ . As the coupled valve opened and the setpoint of TUBE4 stepped from 80 mm to 90 mm at time  $t = 100$  s, the results shown in Fig. 7 were obtained. As shown in Fig. 7a, the water level of TUBE4 tracks its setpoint very slowly owing to the coupling effect between the tubes with conventional PID controllers with fixed parameters. A better performance is shown in Fig. 7b, because the backstepping-PID strategy can obtain more satisfactory PID controller parameters through process model identification and backstepping recursive design.

*Case 2:* The setpoints of TUBE3 and TUBE4 were kept at 80 mm and 90 mm, respectively. The valve V6 opened and a flowrate with 8 mm water level was infused into TUBE4 at time  $t = 500$  s, with the results shown in Fig. 8. It can be seen from the figure that the water levels in the two tubes remain stable after a short time under a large disturbance for these two control schemes.



**Fig. 7** Experimental results for case 1

a Conventional PID controller  
b Backstepping PID strategy



**Fig. 8** Experimental results for case 2

a Conventional PID controller  
b Backstepping PID strategy

**Table 2: Performance comparisons**

Testing	Performance index	Conventional PID	Backstepping-PID
Setpoint tracking	Rise time	98 s	8.5 s
	ISE	5134.6 mm <sup>2</sup> s	999.5 mm <sup>2</sup> s
Disturbance rejection	Maximum error	8.52 mm	8.64 mm
	Settling time	149.5 s	198.5 s

Performance comparisons between the proposed method and the conventional well-tuned PID approach are shown in Table 2. For setpoint changes, the rise time with the proposed method is much shorter than that with conventional PID, and the integrated square error (ISE) with the proposed method is smaller than that with conventional PID. For large disturbances the maximum error with two methods does not differ greatly, and the settling time with the proposed method is a little longer than that with conventional PID. Comparing the conventional PID controller with the proposed backstepping-PID strategy, the setpoint tracking performance of the latter is quite remarkable. Faster responses, weaker coupling and robustness to disturbance can be achieved through online process model identification and PID controller parameter detuning with a backstepping recursive design.

## 5 Conclusions

A concise backstepping-based decentralised PID control scheme has been presented for linear MIMO processes. To reduce computational demand and to avoid potential dangers possibly induced by centralised control schemes, the input and output variables were paired according to RGA and NI rules, and subcontrollers can be efficiently designed flexibly in a parallel prototype. The key feature is that the decentralised controller is equivalent to those designed by the backstepping approach. Furthermore, the backstepping-based decentralised PID control approach can be used in the context of adaptive control to deal with process dynamic variations. As a complementary support to the design procedure, a sufficient condition for the stability of the whole closed-loop system has been analysed using the small-gain theorem, and has been shown that the process tracking performance is improved. The scheme has been tested on the Shell benchmark control problem, and has shown good control performance. An experiment on a real-time water-level control system has demonstrated that the proposed backstepping-PID strategy is able to improve the dynamic performance of the system and reject disturbances rapidly. Further studies on the expansion of the proposed scheme to the adaptive case and to the control of general lower-triangular nonlinear or accurately linearisable processes are underway.

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