A MIXED LOGIC ENHANCED MULTI-MODEL SWITCHING PREDICTIVE CONTROLLER FOR NONLINEAR DYNAMIC PROCESS

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Abstract

In this study a procedure to design multiple model switching predictive controllers (MMSPC) is proposed for the nonlinear dynamic processes with large operation regions. To facilitate the MMSPC design, a general mixed logic dynamic system (MLDS) model is proposed for approximating the nonlinear processes. A major contribution of this study is to integrate a number of techniques to form a novel procedure, and therefore to make multistep state and output predictions effectively realizable within the frame of multiple model switching control. A case for support is presented to demonstrate the efficiency of the design procedure.

Key Words

Nonlinear model predictive control, multiple model switching predictive control, mixed logic, mixed integer quadratic programming

1. Introduction

Predictive control has been an important tool to operate multiple variable and constrained dynamic systems, particularly for process industries such as petroleum and chemical operations. It has been widely agreed that most practical dynamic processes have large operating regions, nonlinearities, and various constraints, especially in chemical and power industries. Therefore, to guarantee the control performance alternative approaches should be considered rather than roughly using ordinary linear model-based predictive control. Nonlinear model based predictive control has been one of the choices to handle this class of processes. However, although a nonlinear model predictive controller is applied to the nonlinear systems directly, the incoming optimization is usually non-convex and the computing effort for the corresponding nonlinear programming becomes very large, making it hard to meet the real-time control requirement [1]. As a simple and effective method, model predictive control with multiple model scheme has been applied for such nonlinear processes, because the processes can be described by a set of linear submodels near different operating points (equilibriums), although the overall process behaviour is still nonlinear [2–5]. Furthermore, many means have been applied to obtain the multiple model descriptions for nonlinear processes with large operating regions, such as fuzzy satisfactory clustering (FSC) [5].

To date two problems have not been properly addressed in multiple model predictive control:

1. Model weighting schemes are not efficient [6–9]. The authors will try to improve the performance in another, future paper.
2. Model switching scheme cannot be directly integrated with predictive control, which prediction impossibility is caused by the principle of model predictive control. The purpose of the current study is to remove these limitations to develop an efficiently feasible procedure to design multiple model switching predictive controllers. Technically, through extension of a mixed logic dynamic system (MLDS) model, which was developed to accommodate integrating logic, dynamics, and constraints [10–12], a mixed logic enhanced multiple switching model is developed. Then, it is possible to obtain the prediction of dynamics of nonlinear processes. It has been noticed that each piecewise linear submodel is an effective representative in a given region for some nonlinear processes with several operating points [3, 4]. Therefore it is not unreasonable to set up logic rules to represent such switching situations.

The contents of this work are organized as follows. In Section 2 a general mixed logic enhanced multiple model description is presented for approximating nonlinear processes with large-scale operating regions. This is a solid basis to make predictive control realizable. In Section 3 a multi-model predictive controller is designed and the

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corresponding stability analysis is developed. Also, the control algorithm is listed step by step as a useful reference for applications. In Section 4 a case study is simulated to demonstrate the effectiveness of the design procedure. In Section 5 we summarize this study and explain further expansions to current studies.

2. A General Mixed Logic Enhanced Multiple Model Description for MMSPC Design

As noted above, direct switching scheme cannot be directly used in MMSPC. Therefore indirect approaches are proposed to resolve this problem. It is well known that only one model is valid ("on") at some future time instant for switching schemes, and other models are not (they are "off"). When time coordinate changes to the next instant, it is required to choose a model from its model set. As when one model is "on" the other models must be "off," the "on" and "off" switching variables can be referred as a binary logic (0 or 1). Thus a model-switching scheme can be treated as a logic relationship. If the multiple modelling of nonlinear processes includes the logic switching conditions, MMSPC can be used to deal with the model prediction course and optimization. The model switching mechanism can be interpreted as IF-THEN rules by introducing model switching conditions. Logic-dependent techniques have been rapidly developed, including logic-based optimization technique [10], design methodology [11], and modelling technique [12]. There have been many applications reported in control and optimization with logic-independent methodologies. Furthermore, these ideas were extended in using the MLDS framework, which has provided a foundation to represent processes operated by interdependent physical laws, logical rules, and operating constraints [12].

In this study a nonlinear process is approximated with a multiple model combined with the logic-switching rule. The process can be classified within several regions with its logic conditions. In fact, the logic rule is a class of IF-THEN rules. For example, if output $y(k+i/k)$ belongs to the valid space of Σ_i, then the controller design is based on the model Σ_i. With the introduction of logic relationships, the original process is described with a hybrid dynamic model, simply because it contains not only continuous variables but logic variables. In order to set up the hybrid model, it starts from multiple model description.

2.1 Piecewisely Linearized Models

Consider a nonlinear dynamic process Σ:

$$\Sigma = \begin{cases} x(k+1) = f(x(k), u(k)) \\ y(k+1) = g(x(k), u(k)) \end{cases} \tag{1}$$

where $u(k) \in R$, $x(k) \in R$, and $y(k) \in R$ denote the control input, state, and output of the plant at time instant k respectively. $f(\cdot)$ and $g(\cdot)$ are smooth nonlinear functions with respect to their variables. For a selected output $y = y_0$, assume that the solution of the equation:

$$\begin{cases} x = f(x, u) \\ y = g(x, u) = y_0 \end{cases}$$

exists; then (x_0, y_0, u_0) is an equilibrium of the process. Note that the equilibrium really exists for most nonlinear processes with large operating regions. This assumption is a necessary condition for multi-model control scheme. For a nonlinear process with several operating points/equilibriums (x_i, y_i, u_i), linearization of the nonlinear process around these operating points will give rise to a bank of piecewise linear models. More analytical discussion and verification of the piecewise linearization techniques around the equilibrium can be found in [3, 4].

Now, suppose there exist S outputs in the output space Y, which satisfies $y_{\min} \leq y_0 \leq \cdots \leq y_{S-1} \leq y_{\max}$. Points $(x_i, y_i, u_i), i = 0, \ldots, S-1$ are equilibriums of the nonlinear process. Linear submodels are obtained by linearizing the nonlinear system at their respective equilibrium points. For example a submodel Σ_i:

$$\Sigma_i = \begin{cases} x(k+1) = \frac{\partial f}{\partial x}\big|_{(x_i,u_i)} x(k) + \frac{\partial f}{\partial u}\big|_{(x_i,u_i)} u(k) - b_x \\ y(k) = \frac{\partial g}{\partial x}\big|_{(x_i,u_i)} x(k) + \frac{\partial g}{\partial u}\big|_{(x_i,u_i)} u(k) - d_y \\ b_x = \frac{\partial f}{\partial x}\big|_{(x_i,u_i)} x_i + \frac{\partial f}{\partial u}\big|_{(x_i,u_i)} u_i - x_i \\ d_y = \frac{\partial g}{\partial x}\big|_{(x_i,u_i)} x_i + \frac{\partial g}{\partial u}\big|_{(x_i,u_i)} u_i - y_i \end{cases} \tag{2}$$

Because of the local property of each linear submodel, the nonlinear process can be expressed approximately with the linear model Σ_i around the equilibrium (x_i, y_i, u_i), and also Σ_i is applicable for the output space $y_i \pm \alpha_i$. With reference to (2), the whole nonlinear process can be represented in the following condensed form:

$$x(k+1) = \begin{cases} A_0 x(k) + B_0 u(k) - b_0 \\ \vdots \\ A_{S-1} x(k) + B_{S-1} u(k) - b_{S-1} \end{cases}$$

$$y(k) = \begin{cases} C_0 x(k) + D_0 u(k) - d_0 \\ \vdots \\ C_{S-1} x(k) + D_{S-1} u(k) - d_{S-1} \end{cases}$$

2.2 A Mixed Logic Based Model-Switching Rule

As a matter of fact, model switching rule is a boundary classifier to govern the mixed logic multimodel structure. To denote the qualitative relationships of a nonlinear process, a logic variable $\delta_i \in \{0, 1\}, (i = 0, \ldots, S-1)$ is introduced, which is an indicator that designates the linear submodel at each time instant. The logic variables and process outputs have the following equivalent relationships:

$$\begin{aligned}
&\delta_0 = 1 \leftrightarrow y \le y_0 + \alpha_0 \\
&(\delta_0 = 0 \wedge \delta_1 = 1) \leftrightarrow y \le y_1 + \alpha_1 \\
&\quad\vdots \\
&(\delta_0 = 0 \wedge \delta_1 = 0 \wedge \cdots \wedge \delta_{S-1} = 1) \leftrightarrow y \le y_{S-1} + \alpha_{S-1}
\end{aligned} \tag{3}$$

Define $M_{yi} \triangleq \max\limits_{x \in X}(y - y_i - \alpha_i)$, $m_{yi} \triangleq \min\limits_{x \in X}(y - y_i - \alpha_i)$, and ε is a small tolerance, normally, $\varepsilon = 10^{-6}$.

Here introduce $S-1$ auxiliary logical variables to denote the logical conditions:

$$\begin{aligned}
&(\delta_0 = 0 \wedge \delta_1 = 1) \leftrightarrow \delta'_1 = 1 \\
&(\delta_0 = 0 \wedge \delta_1 = 0 \wedge \delta_2 = 1) \leftrightarrow \delta'_2 = 1 \\
&\quad\vdots \\
&(\delta_0 = 0 \wedge \delta_1 = 0 \wedge \cdots \wedge \delta_{S-1} = 1) \leftrightarrow \delta'_{S-1} = 1
\end{aligned}$$

Then relationships of (3) can be equivalently expressed as:

$$\begin{aligned}
&y - y_0 - \alpha_0 \le M_{y0}(1 - \delta_0) \\
&y - y_0 - \alpha_0 \ge \varepsilon + (m_{y0} - \varepsilon)\delta_0 \\
&\quad\vdots \\
&y - y_{S-1} - \alpha_{S-1} \le M_{yS-1}(1 - \delta'_{S-1}) \\
&y - y_{S-1} - \alpha_{S-1} \ge \varepsilon + (m_{yS-1} - \varepsilon)\delta'_{S-1}
\end{aligned}$$

Because the multiple model description for the nonlinear process satisfies the exclusive-or logic condition, an additional equation is introduced:

$$\sum_{i=0}^{S-1} \delta_i = 1$$

2.3 The Mixed Logic-Based Multiple Model

The nonlinear process can be expressed in terms of the multiple linear submodels plus the logic conditions:

$$x(k+1) = \begin{cases} A_0 x(k) + B_0 u(k) - b_0 & \text{if } \delta_0(k) = 1 \\ \vdots & \vdots \\ A_{S-1} x(k) + B_{S-1} u(k) - b_{S-1} & \text{if } \delta_{S-1}(k) = 1 \end{cases}$$

$$y(k) = \begin{cases} C_0 x(k) + D_0 u(k) - d_0 & \text{if } \delta_0(k) = 1 \\ \vdots & \vdots \\ C_{S-1} x(k) + D_{S-1} u(k) - d_{S-1} & \text{if } \delta_{S-1}(k) = 1 \end{cases}$$

Each submodel can be associated to a logic variable δ_i such that:

$$[\delta_i = 1] \leftrightarrow \begin{cases} x(k+1) = A_i x(k) + B_i u(k) - b_i \\ y(k) = C_i x(k) + D_i u(k) - d_i \end{cases}$$

Then the nonlinear process can be rewritten as:

$$x(k+1) = \sum_{i=0}^{S-1} [A_i x(k) + B_i u(k) - b_i]\delta_i(k) \tag{4}$$

$$y(k) = \sum_{i=0}^{S-1} [C_i x(k) + D_i u(k) - d_i]\delta_i(k) \tag{5}$$

Obviously (4) and (5) are nonlinear equations because of the existence of the product operation on the right-hand side of the expressions. The product terms can be replaced by auxiliary variables $z_{i1}(k)$ and $z_{i2}(k)$; then (4) and (5) are linear as:

$$x(k+1) = \sum_{i=0}^{S-1} z_{i1}(k), \quad y(k) = \sum_{i=0}^{S-1} z_{i2}(k)$$

$z_{i1}(k)$ and $z_{i2}(k)$ are defined as:

$$\begin{aligned}
&z_{i1}(k) \triangleq [A_i x(k) + B_i u(k) - b_i]\delta_i(k), \\
&z_{i2}(k) \triangleq [C_i x(k) + D_i u(k) - d_i]\delta_i(k), \quad i = 0, \ldots, S-1.
\end{aligned}$$

In order to set up the linkage between the continuous variables and logic variables, it is necessary to estimate the over and under bound of the continuous variable. Define:

$$\begin{aligned}
M_1 &= [M_{11}, \ldots, M_{1n}], \quad m_1 = [m_{11}, \ldots, m_{1n}], \\
M_2 &= [M_{21}, \ldots, M_{2n}], \quad m_2 = [m_{21}, \ldots, m_{2n}]
\end{aligned}$$

$$\begin{aligned}
M_{1j} &= \max_{i=0,\ldots,S-1} \{\max A_i^j x + B_i^j u - b_i^j\}, \\
m_{1j} &= \min_{i=0,\ldots,S-1} \{\max A_i^j x + B_i^j u - b_i^j\}
\end{aligned}$$

$$\begin{aligned}
M_{2j} &= \max_{i=0,\ldots,S-1} \{\max C_i^j x + D_i^j u(k) - d_i^j\}, \\
m_{2j} &= \min_{i=0,\ldots,S-1} \{\max C_i^j x + D_i^j u(k) - d_i^j\}
\end{aligned}$$

The definitions of the auxiliary variable $z_{i1}(k)$ and $z_{i2}(k)$ are equivalent to the following inequalities:

$$\begin{aligned}
&z_{il}(k) \le M_l \delta_i(k) \\
&z_{il}(k) \ge m_l \delta_i(k) \\
&z_{il}(k) \le A_i x(k) + B_i u(k) - b_i - m_l(1 - \delta_i(k)) \\
&z_{il}(k) \ge A_i x(k) + B_i u(k) - b_i - M_l(1 - \delta_i(k))
\end{aligned}$$

where $l = 1, 2$.

Then the nonlinear process can be transformed into a linear mixed logic model:

$$\begin{cases} x(k+1/k) = A_k x(k/k) + B_{1k} u(k/k) \\ \qquad + B_{2k}\delta(k/k) + B_{3k} z(k/k) + B_{4k} \\ y(k/k) = C_k x(k/k) + D_{1k} u(k/k) + D_{2k}\delta(k/k) \\ \qquad + D_{3k} z(k/k) + D_{4k} \\ E_{2k}\delta(k+i/k) + E_{3k}\delta(k+i/k) \\ \quad \leq E_{1k} u(k+i/k) + E_{4k} x(k+i/k) + E_{5k} \end{cases} \tag{6}$$

where $A_k, B_{1k}, \ldots, B_{4k}, C_k, D_{1k}, \ldots, D_{4k}, E_{1k}, \ldots, E_{5k}$ can be derived from the aforementioned logic relation description directly.

3. Design of Multi-Model Predictive Controller

In general, model predictive control is composed of two parts, state and output prediction and receding optimization.

3.1 State and Output Prediction

The mixed logic dynamic model of (6) is a linear expression to describe the whole process. Similarly with linear state space model, the future state and output can be predicted by iterating the mixed logic dynamic model. For simplicity, but not losing generality, control horizon and prediction horizon are set up with an equal duration T. Accordingly the model state prediction course is determined as:

$$\begin{aligned} x(k+1/k) &= A_k x(k/k) + B_{1k} u(k/k) + B_{2k}\delta(k/k) \\ &\quad + B_{3k} z(k/k) + B_{4k} \\ x(k+2/k) &= A_k x(k+1/k) + B_{1k} u(k+1/k) \\ &\quad + B_{2k}\delta(k+1/k) \\ &\quad + B_{3k} z(k+1/k) + B_{4k} \\ &\vdots \qquad\qquad \vdots \\ x(k+T-1/k) &= A_k x(k+T-2/k) \\ &\quad + B_{1k} u(k+T-2/k) \\ &\quad + B_{2k}\delta(k+T-2/k) \\ &\quad + B_{3k} z(k+T-2/k) + B_{4k} \\ x(k+T/k) &= x_f \end{aligned} \tag{7}$$

Similarly, the output $y(k+i/k)$ predictions can be obtained in the same way. In (7) set $x(k+T/k) = x_f$ as a terminal equality constraint that can guarantee the stability of the system. It should be mentioned that the horizon T is an important parameter in the control system design. Increasing the horizon T can make optimization feasible, but on the other hand the computational burden also increases greatly.

3.2 Receding Horizon Optimization

The objective of predictive control based on the mixed logic is consistent with that from classical model predictive control. For a given initial state (x_0, y_0), desired final state (x_f, y_f), and optimization horizon T, the control sequence $u_0^{T-1} = \{u(0), u(1), \ldots, u(T-1)\}$ is to be searched to satisfy the following cost function:

$$\begin{aligned} J(u_0^{T-1}, x_0)) = \sum_{t=0}^{T-1} &\|u(k) - u_f\|_{Q_1}^2 + \|\delta(k, x_0, u_0^t) - \delta_f\|_{Q_2}^2 \\ &+ \|z(k, x_0, u_0^t) - z_f\|_{Q_3}^2 + \|x(k, x_0, u_0^t) \\ &- x_f\|_{Q_4}^2 + \|y(k, x_0, u_0^{t-1}) - y_f\|_{Q_5}^2 \end{aligned} \tag{8}$$

where $\|x\|_Q^2 = x'Qx, Q_i = Q_i' \geq 0, i = 1, \ldots, 5$ are positive or semi-positive matrices.

The process is subjected to the equality and inequality constraints. The equality constraints specify terminal conditions, and the system equation can be expressed as:

$$\begin{aligned} x(T, x_0, u_0^{T-1}) &= x_f \\ x(k+1/k) &= A_k x(k/k) + B_{1k} u(k/k) + B_{2k}\delta(k/k) \\ &\quad + B_{3k} z(k/k) + B_{4k} \\ y(k/k) &= C_k x(k/k) + D_{1k} u(k/k) + D_{2k}\delta(k/k) \\ &\quad + D_{3k} z(k/k) + D_{4k} \end{aligned} \tag{9}$$

The inequality constraint can also be rewritten following standard form:

$$\begin{aligned} &E_{2k}\delta(k+i/k) + E_{3k}\delta(k+i/k) \\ &\quad \leq E_{1k} u(k+i/k) + E_{4k} x(k+i/k) + E_{5k} \end{aligned} \tag{10}$$

where $i = 0, \ldots, T-1$.

Let $\{u_k^*(k+i)\}_{i=0,\ldots,T-1}$ be the optimum of control sequence at current time instant; from the receding horizon policy of the predictive control, set:

$$u(k) = u_k^*(0) \tag{11}$$

Remark 1. Model predictive control (MPC) is based on optimization, so the feasibility of optimal problem is a necessary condition for implementing the control algorithm. On the other hand, horizon T represents the control degree of freedom in MPC, that is, the control solution is a sequence $u(0), \ldots, u(T-1)$. In this study, terminal equality constraint is imposed on the optimization problem, which is used to guarantee the stability of closed-loop process. Herein, to design the controller requires the calculation of a constrained optimal problem, at each time instant, with satisfaction of the terminal equality constraint. However, for a process with large time coefficient or large time delay while the current state has departed from the origin, it is impossible to drive the current state to the origin exactly through the control sequence $(u(0), \ldots, u(T-1))$ within a short duration (i.e., T is small). In the other words, small horizon T will make the optimization problem infeasible. In fact, the horizon T cannot be selected until the optimization problem is feasible. It has been proven that there exists a finite horizon length above which a receding horizon policy provides both feasibility and stability [13]. Another method [14] that emerged in the study is to resolve the optimization problem without terminal constraint condition, and then judge whether the terminal constraints are satisfied or not. If they are not, the horizon can be increased until it is satisfied.

3.3 An Optimization Procedure

From Section 3.2, we can notice that the optimization is a mixed integer quadratic programming problem for dealing with MLDS. Generally the decision variables of predictive controller for MLDS are made up of three parts: control input sequence u, logic variable sequence δ, and the auxiliary variable sequence z. In this study a solution for this optimization problem is found in [12], and is outlined as follows.

First predict the process states and outputs at time instant $k+t$ by the following formulation:

$$\begin{aligned} x(k+t/k) = A_k^t x_0 + \sum_{i=0}^{t-1} A_k^i \times [&B_{1k} u(k+t-1-i/k) \\ &+ B_{2k}\delta(k+t-1-i/k) \\ &+ B_{3k} z(k+t-1-i/k) + B'_{4k}] \end{aligned} \tag{12}$$

Substitute (12) into (7), and define the following vector:

$$\Omega = \begin{bmatrix} u(0) \\ \vdots \\ u(T-1) \end{bmatrix}, \quad \Delta = \begin{bmatrix} \delta(0) \\ \vdots \\ \delta(T-1) \end{bmatrix},$$

$$\Xi = \begin{bmatrix} z(0) \\ \vdots \\ z(T-1) \end{bmatrix}, \quad \upsilon = \begin{bmatrix} \Omega \\ \Delta \\ \Xi \end{bmatrix}$$

Then resolve the equivalent standard mixed-integer quadratic programming (MIQP) problem [10]:

$$\begin{cases} \min \upsilon' S_1 \upsilon + 2(S_2 + x_0' S_3)\upsilon \\ F_1 \upsilon \le F_2 + F_3 x_0 \end{cases}$$

where $S_i, F_i, i=1,2,3$ can be derived from (12).

This is a mixed integer quadratic programming problem with the performance index (8), and constraints (9) and (10), which can be resolved by MIQP Matlab toolbox [15, 16].

3.4 Stability Analysis of the Control Algorithm

Assume that a local model can accurately represent its process dynamics in the absence of disturbance, and that there is a theorem to guarantee the asymptotic stability of the closed-loop process.

Theorem 1. Let (x_e, u_e) be equilibrium of the nonlinear process and (δ_e, z_e) be an admissible pair by introducing the logic variables and auxiliary variables. $x(0)$ is such an initial state that (8) is feasible and $(x_e, u_e, \delta_e, z_e, y_e)$ is the terminal state of the controlled process. Then when there is no disturbance and the controller parameters satisfy $Q_1 > 0$, $Q_2 \ge 0$, $Q_3 \ge 0$, $Q_4 > 0$, $Q_5 \ge 0$, MMSPC control law (8–11) can stabilize the nonlinear process.

Proof. Let $V(x(k)) = V(x,k,k+T-1)$ denote the optimal value of the cost functional whose time horizon is from k to $k+T-1$, and let $V^*(x(k)) = V(x,k+1, k+T-1)$ denote the optimal value of the cost functional whose time horizon is from $k+1$ to $k+T-1$.

For the optimization of the objective function (8), let U_k^* denote the optimal control sequence $\{u_k^*(0),\ldots,u_k^*(T-1)\}$, and let $V(x(k)) = J(U_k^*, x(k))$. When $x(k) = \xi$, the optimal value of the cost functional is $V(\xi, k, k+T-1)$.

For the closed-loop process (1), suppose that $f_c(\xi) = f(\xi, u_k^*(0))$. From the cost functional (8), we can obtain:

$$\begin{aligned} V(\xi) = &\|u(k) - u_e\|_{Q_1}^2 + \|\delta(k) - \delta_e\|_{Q_2}^2 + \|z(k) - z_e\|_{Q_3}^2 \\ &+ \|\xi - \xi_e\|_{Q_4}^2 + \|y(k) - y_e\|_{Q_5}^2 + V^*(f_c(\xi)) \end{aligned}$$

Then $V(\xi) > V^*(f_c(\xi))$ is proved.

At time instant $k+1$, control $u(\tau)$ $(k+1 \le \tau \le k+T-1)$ is a optimal sequence when optimization problem is resolved over $[k+1, k+T-1]$. Because there is no disturbance in the process, the optimal value at current time instant is $V^*(f_c(\xi))$, that is, $u(\tau) = \{u_k^*(0),\ldots,u_k^*(T-1)\}$. Furthermore $\{u_k^*(0),\ldots,u_k^*(T-1)\}$ is the optimal solution of the objective function (8), and therefore this control sequence can achieve $x(T) = x_e$. As (δ_e, z_e) is the admissible pair of the process, it has $\delta(T) = \delta_e, z(T) = z_e$, and $y(T) = y_e$. Additionally, (x_e, u_e) is the equilibrium pair of the process, so $x_e = f(x_e, u_e)$. Then $\{u_k^*(0),\ldots,u_k^*(T-1), u_e\}$ is a feasible control sequence obtained by minimizing the cost function $V(f_c(\xi))$ over $[k+1, k+T]$. For the minimization problem $V(f_c(\xi))$ with one feasible solution $V^*(f_c(\xi))$, we have $V^*(f_c(\xi)) \ge V(f_c(\xi))$ by optimality.

As $V(\xi) > V^*(f_c(\xi))$, we have $V(\xi) > V(f_c(\xi))$, that is, $V(x(k)) > V(x(k+1))$. $V(k)$ is a *Lyapunov* function having attenuation monotonicity property, and according to the *Lyapunov* stability theorem, the MMSPC control law can ensure the closed-loop asymptotic stability at the process equilibrium. □

3.5 Algorithm Steps

In total, the multiple model switching predictive control can be implemented by following a sequence of events during every sampling interval, with the process to be controlled being sampled periodically:

1. Partition the nonlinear system model into S linear models as in (2).
2. Define mixed logic based model-switching rule by (3).
3. Set up the MLD model for the system.
4. Sample the process output (time instant t) and predict the future output with the MLD model.
5. Obtain the optimum control action by MIQP.
6. Implement the control action to the process.
7. Wait for sampling clock pulse; then go to step 4.

Remark 2. The linear model that is obtained by linearization of the nonlinear system around some operating point is an approximation to the real system and only effective within a neighbourhood of the operating point. The scale of the neighbourhood depends on the nonlinearity of the process. Generally, the more strongerly nonlinear the process is, the smaller the scale should be designated. Therefore, when more linear models are used, the precision of an MLDS model that represents the nonlinear process will be improved. However, the computational burden will be exponentially increased along with the number of linear models (i.e., logic variables) because MIQP is an NP-hard problem [10]. In this way, one has to settle for a compromise between the control effect and computational burden. In fact, desired operating points are limited for practical implementation, that is, account of linear models is also limited. For example, six linear models are enough to represent a two-input two-output strongly nonlinear process.

4. Simulation Studies

To validate the control design procedure, consider a nonlinear process described by:

$$\begin{cases} x_1(k+1) = -0.25x_2(k) \\ x_2(k+1) = x_1^2(k) + 1.3x_2(k) + u(k) \\ y(k) = x_1(k) \end{cases}$$

The initial state is $x_1(0)=0$, $x_2(0)=0$, and input $u = 0$. The set point of the system is $y_r = 3$.

The input, state, and output are, respectively, bounded by:

$$U = \left\{ u \in R^1 \middle| -8 \leq x \leq 0 \right\},$$

$$X = \left\{ (x_1, x_2) \in R^2 \middle| \begin{array}{l} -4 \leq x_1 \leq 4 \\ -16 \leq x_2 \leq 16 \end{array} \right\},$$

$$Y = \left\{ y \in R^1 \middle| -4 \leq y \leq 4 \right\}$$

To approximate the nonlinear model, three linear submodels were partitioned at points $y_0 = 0$, $y_1 = 1$, and $y_2 = 2$, which were formulated as:

$$\#0\colon \begin{cases} x_1(k+1) = -0.25x_2(k) \\ x_2(k+1) = 1.3x_2(k) + u(k) \\ y(k) = x_1(k) \end{cases}$$

$$\#1\colon \begin{cases} x_1(k+1) = -0.25x_2(k) \\ x_2(k+1) = 2x_1(k) + 1.3x_2(k) + u(k) - 1 \\ y(k) = x_1(k) \end{cases}$$

$$\#2\colon \begin{cases} x_1(k+1) = -0.25x_2(k) \\ x_2(k+1) = 4x_1(k) + 1.3x_2(k) + u(k) - 4 \\ y(k) = x_1(k) \end{cases}$$

The model-switching rules were assigned as:

$$\#0\colon y < 0.5;$$
$$\#1\colon 0.5 \leq y < 1.5;$$
$$\#2\colon y \geq 1.5$$

Three logic variables δ_0, δ_1, and δ_2 and two auxiliary logic variables δ_1' and δ_2' were required to designate the models. Note that the models have the feature $y = x_1(k)$; hence some auxiliary output variables can be omitted. Therefore only the auxiliary variables were defined for the states z_0, z_1, and z_2.

After calculation, we had $M_1 = \begin{bmatrix} 4 \\ 20.8 \end{bmatrix}$, $m_1 = \begin{bmatrix} 4 \\ 16.8 \end{bmatrix}$. The horizon parameter $T = 3$ was chosen to facilitate the computation of mixed logic quadratic program. The simulation results are shown in Fig. 1. It is clear that MMSPC, based on the mixed logic, is effective in controlling the nonlinear process. The output is stabilized at the set point with satisfaction of the constraints.

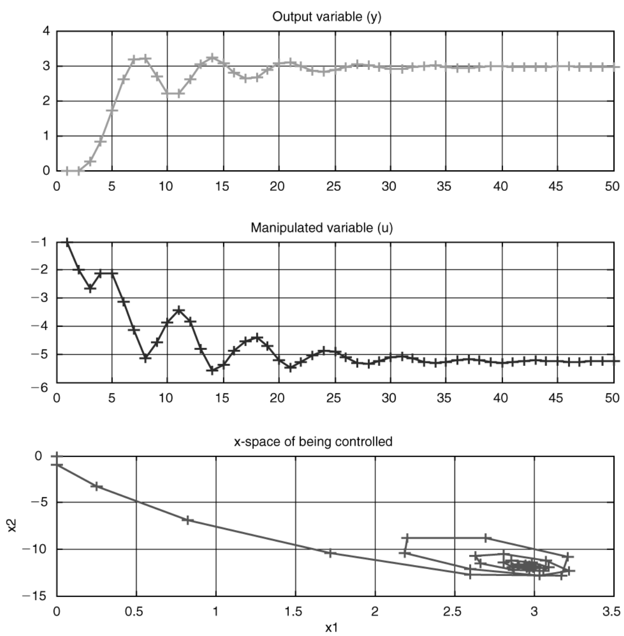

Figure 1. Multi-model switching predictive control for a nonlinear system.

5. Conclusion

An obvious advancement from this newly designed controller is to remove the impossibility of state and output prediction in MMSPC applications. We begin by justifying the problem using multiple model description for predictive control, which has been the dominant barrier against state and output prediction in nonlinear predictive control. An alternative modelling technique with switching strategy for a nonlinear process with several operating points is presented to make the predictions efficiently feasible; therefore a prediction control system is designed. Linearization of the nonlinear process around its operating points gives a bank of piecewise linear systems. Each linear submodel covers its corresponding operating region in state space, in which the whole model provides a reasonable approximation to the nonlinear process. Combined with logic variables, these linear submodels can form an MLDS description to represent the nonlinear process globally.

Rather than developing a completed new theory and algorithm, an exemplary insight philosophically considered in the study is that a clever crafter of some well-developed methodologies can significantly advance work on difficult problems.

The stability analysis and step-by-step implementation of the control algorithm provide useful references for designers. A simulated bench test has been carried out to demonstrate the efficiency of the design procedure. We plan to further investigate certain representative cases from laboratory experimental rigs to in-field applications. A large scale of bench tests will be reported in follow-up studies.

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