

# Decentralized PD Control for Non-uniform Motion of a Hamiltonian Hybrid System

Mingcong Deng<sup>1,\*</sup>    Hongnian Yu<sup>2</sup>    Akira Inoue<sup>1</sup>

<sup>1</sup>Department of Systems Engineering, Okayama University, 3-1-1 Tsushima-Naka, Okayama 700-8530, Japan

<sup>2</sup>Faculty of Computing, Engineering and Technology, Staffordshire University, Stafford ST18 0DG, UK

---

**Abstract:** In this paper, a decentralized proportional-derivative (PD) controller design for non-uniform motion of a Hamiltonian hybrid system is considered. A Hamiltonian hybrid system with the capability of producing a non-uniform motion is developed. The structural properties of the system are investigated by means of the theory of Hamiltonian systems. A relationship between the parameters of the system and the parameters of the proposed decentralized PD controller is shown to ensure local stability and tracking performance. Simulation results are included to show the obtained non-uniform motion.

**Keywords:** Decentralized proportional-derivative (PD) control, hybrid system, non-uniform motion, local stability, tracking performance.

---

## 1 Introduction

In industrial applications such as programmable cutting, the cutting system is required to have extreme acceleration characteristics to produce a clean cut in the material. However, the range and speed of the obtainable output motions are limited by certain motor characteristics, such as the motor's rated torque, peak power capability and bandwidth. Meanwhile, the torque requirement from the motor during excessive torque fluctuations is to produce substantial heat generation in the motor windings. To save energy and reduce the servomotor's cost, hybrid linkage systems were considered by using a small servomotor and a constant velocity motor. For the design of a hybrid linkage system, a lot of work has been done by many researchers. A hybrid seven-bar mechanical design without considering the mathematical model was developed in [1]. The kinematics optimization of a hybrid seven-bar system was also studied in [2]. These seven-bar systems were parallel with the study of a hybrid linkage system without sliding output motion for a clean cut in [3]. Recently, a hybrid linkage system offering an alternative means to generate a programmable third independent rotation and sliding output motion was designed in [4]. However, the control system design for the hybrid system was not considered.

The purpose of the work is as follows. The hybrid system in [4] is investigated based on Hamiltonian formulation, because the main advantage of the Hamiltonian formulation is the storage function of the passive map being precisely the total energy of the closed-loop system, and the output passivity can be guaranteed by a so-called damping injection gain (differential compensation)<sup>[5]</sup>. By using the hybrid linkage system, a non-uniform motion

(nonlinear motion) is obtained by combining the independent rotational motion and the sliding output motion. For control of the non-uniform motion, a decentralized proportional-derivative (PD) controller is designed by extending the general PD controller structure<sup>[6,7]</sup> to the Hamiltonian system. Based on the properties of the Hamiltonian system, a relationship between the parameters of the hybrid system and the parameters of the decentralized PD controller is shown to ensure local stability and to evaluate tracking performance. Simulation results are included to show the obtained non-uniform motion.

### Notations

$K_e$  : The total kinetic energy.

$P_e$  : The potential energy of the system.

$\alpha_k$  : The angle with respect to the reference position of the link parallel and having same direction with  $x$  coordinate ( $k = 1, \dots, 8$ ).

$I_i$  : The moment of inertia of motor armature ( $\text{kg} \cdot \text{m}^2$ ), moment of inertia of load ( $\text{kg} \cdot \text{m}^2$ ), and moment of inertia of linkages ( $\text{kg} \cdot \text{m}^2$ ). When  $i = 1, 4, 5$ ,  $I_i$  denotes the moment of inertia of link  $I_i$  about the ground. When  $i = 2, 3, 7, 8$ ,  $I_i$  denotes the moment of inertia of link  $I_i$  about its centre of mass.

$L_i$  : The linkage of the hybrid linkage system.

$x_i$  :  $x$  coordinate of mass center of linkages  $L_2, L_3, L_4, L_6, L_7$ , and  $L_8$  (m).

$y_i$  :  $y$  coordinate of mass center of linkages  $L_2, L_3, L_4, L_6, L_7$ , and  $L_8$  (m).

$m_i$  : The linkage mass (kg).

$T_k$  : The generalized force or torque associated with the generalized coordinate  $\alpha_k$  (angular displacement (rad),  $k = 1, \dots, 8$ ).

---

Manuscript received January 15, 2007; revised December 20, 2007

\*Corresponding author.

E-mail address: deng@suri.sys.okayama-u.ac.jp

$T_{\text{servo}}$  : The torque of the servo motor to drive crank (N · m).

$T_{\text{cv}}$  : The torque of the constant velocity motor to drive crank (N · m).

$T_{\text{output}}$  : The output torque of link 4.

$f_2$  : The initial distance between the slider start point and the servo motor fixed pivot.

$f_1$  : The distance between the constant velocity motor fixed pivot and the servo motor fixed pivot.

$f_0$  : The maximum of the required slider motion.

$\gamma$  : The initial angular displacement.

$\theta$  :  $\theta = \alpha_2 - \alpha_7$ .

$T$  : The time interval (s).

## 2 Problem setup and Hamiltonian dynamics of the hybrid system

The configuration of the hybrid linkage system driven by a constant velocity motor and a servomotor is shown in Fig. 1. The system has eight links,  $L_1, \dots, L_8$ . It is noted that links 2, 7, and 8 are fixed together, i.e., there is no relative movement among them. For a given set of linkage dimensions, the mechanism requires two independent inputs,  $\alpha_1$  and  $\alpha_5$ , to produce specified rotational output variable  $\alpha_4$  and sliding output motion  $f(t)$ , where  $\alpha_4$  and  $f(t)$  are from tasks. We have set the sliding block in a parallel slot so that the block can only do the parallel motion. In this paper, we set  $f(t) = f_2 + f_0 \sin(\alpha_4 + \gamma)$ . Then, other variables can be decided by the kinematics analysis of the mechanism, when the sliding motion satisfies  $\gamma = \pi/2$ .

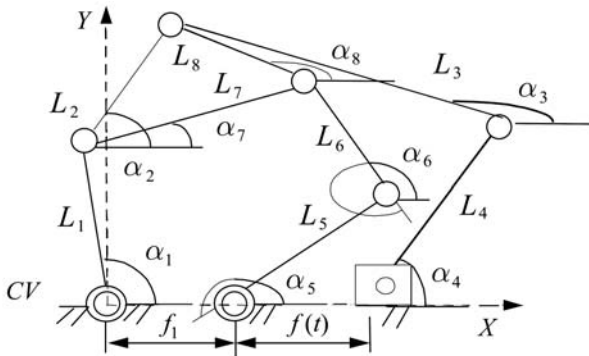


Fig. 1 The hybrid linkage system

Impose the following assumptions on the hybrid linkage system.

### Assumptions.

- 1) The links are rigid.
- 2) Friction in all joints other than 1, 4, and 5 joints (angles) is negligible. Friction of sliding output motion is also negligible.
- 3) There is no backlash in the joints.

The kinetic energy of the system is

$$K_e = \sum_{i=1}^8 \frac{1}{2} I_i \dot{\alpha}_i^2 + \sum_{i=2,3,4,6,7,8} \frac{1}{2} m_i (\dot{x}_i^2 + \dot{y}_i^2) \quad (1)$$

and the potential energy of the system is

$$P_e = \sum_{i=1}^8 m_i g y_i \quad (2)$$

where  $m_8 = 0$ , because we choose arc ( $L_2$  and  $L_7$ ) instead of triangle ( $L_2$ ,  $L_7$ , and  $L_8$ ) in real designs, and

$$x_1 = 0.5L_1 \cos \alpha_1, \quad y_1 = 0.5L_1 \sin \alpha_1$$

$$x_2 = L_1 \cos \alpha_1 + 0.5L_2 \cos(\alpha_7 + \theta)$$

$$y_2 = L_1 \sin \alpha_1 + 0.5L_2 \sin(\alpha_7 + \theta)$$

$$x_3 = L_4 \cos \alpha_4 + 0.5L_3 \cos \alpha_3 + f_1 + f(t)$$

$$y_3 = L_4 \sin \alpha_4 + 0.5L_3 \sin \alpha_3$$

$$x_4 = 0.5L_4 \cos \alpha_4 + f_1 + f(t), \quad y_4 = 0.5L_4 \sin \alpha_4$$

$$x_5 = 0.5L_5 \cos \alpha_5 + f_1, \quad y_5 = 0.5L_5 \sin \alpha_5$$

$$x_6 = L_5 \cos \alpha_5 + 0.5L_6 \cos \alpha_6 + f_1$$

$$y_6 = L_5 \sin \alpha_5 + 0.5L_6 \sin \alpha_6$$

$$x_7 = L_1 \cos \alpha_1 + 0.5L_7 \cos \alpha_7$$

$$y_7 = L_1 \sin \alpha_1 + 0.5L_7 \sin \alpha_7.$$

The Hamiltonian equations of motion are expressed as

$$\dot{q} = \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_5 \end{bmatrix} = \frac{\partial H}{\partial p} \quad (3)$$

$$\dot{p} = -\frac{\partial H}{\partial p} + \begin{bmatrix} T_1 \\ T_5 \end{bmatrix} \quad (4)$$

$$H = \frac{1}{2} \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_5 \end{bmatrix}^T \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_5 \end{bmatrix} + P_e \quad (5)$$

where  $H$  is the Hamiltonian, and

$$M_{11} = J'_{11} + J_{31} - J'_{41}, \quad M_{12} = J'_{12} + J_{32} - J'_{42}$$

$$M_{21} = K'_{11} + K_{31} - K'_{41}, \quad M_{22} = K'_{12} + K_{32} - K'_{42}.$$

The analytical expressions for the coefficients  $K_{ij}$ ,  $J_{ij}$ ,  $K'_{ij}$ , and  $J'_{ij}$  have been obtained by general dynamic analysis in the early work<sup>[4]</sup>.

The mathematical descriptions of the generalized torque are given as follows. In this case, the generalized force includes the applied torque from links  $L_1$ ,  $L_4$ , and  $L_5$ , where  $T_{\text{output}}$  is payload. So we have

$$T_1 = \sum_{i=1,4,5} T'_i \frac{\partial \alpha_i}{\partial \alpha_1} = T_{\text{cv}} + T_{\text{output}} \frac{\partial \alpha_4}{\partial \alpha_1} \quad (6)$$

$$T_5 = \sum_{i=1,4,5} T'_i \frac{\partial \alpha_i}{\partial \alpha_5} = T_{\text{servo}} + T_{\text{output}} \frac{\partial \alpha_4}{\partial \alpha_5} \quad (7)$$

where  $T_1' = T_{cv}$ ,  $T_5' = T_{servo}$ ,  $T_4' = T_{output}$ , and  $T_{output}$  is rotating torque. In this paper, we assume the mass of sliding block to be zero. If  $T_{output}$  is unknown, then this term is regarded as a kind of disturbance. Some physical interpretations reveal that for the acceleration term, the inertia matrix is symmetric, positive definite, and configuration-dependent. It can be proved that  $M_{12}$  and  $M_{21}$  have the following properties:

- 1)  $J_{12}' = K_{11}'$ ;
- 2)  $J_{32}' - J_{42}' = K_{31}' - K_{41}'$ .

From (5), the following energy balance immediately follows.

$$\frac{d}{dt}H = \frac{\partial^T H}{\partial q} \dot{q} + \frac{\partial^T H}{\partial p} \dot{p} = \dot{q}^T \begin{bmatrix} T_1 \\ T_5 \end{bmatrix}.$$

That is, with input  $[T_1, T_2]^T$  and output derivative of  $q$ , the system is passive, if  $P_e$  is bounded from below. It is easy to prove that  $P_e$  is bounded from the derivative of  $P_e$  with respect to  $q$ <sup>[5]</sup>. To evaluate the proposed hybrid linkage system, the structural relations have been proved.

1)  $m_1 \neq 0$  or  $m_5 \neq 0$  (other  $m_i = 0$ ), the proposed system is equivalent to a one-link arm<sup>[8]</sup>.

2) When  $m_1 \neq 0$ ,  $m_2 \neq 0$  or  $m_5 \neq 0$ , and  $m_6 \neq 0$  (other  $m_i = 0$ ), the proposed system is equivalent to a two-link arm<sup>[9, 10]</sup>.

3) When  $m_1 \neq 0$ ,  $m_7 \neq 0$ ,  $m_5 \neq 0$ , and  $m_6 \neq 0$  (other  $m_i = 0$ ), the proposed system is equivalent to a 4R linkage<sup>[11]</sup>.

4) When  $L_6 = 0$  and  $f(t)$  is constant, the proposed system is equivalent to the Stephenson six-bar linkage<sup>[11]</sup>.

5) When  $L_8 = 0$ , the proposed system is equivalent to a slider hybrid system<sup>[2]</sup>.

To avoid singular values of  $A_1^{-1}$  and  $C_1^{-1}$ , we must set  $\alpha_3 \neq \alpha_4$  and  $\alpha_6 \neq \alpha_7$ . Meanwhile, it should be remembered that the quadratic solutions of the triangle function have to be considered. The desired solution will be determined by the desired linkage position.

### 3 Parameter design of the proposed PD controller

In this section, a decentralized PD controller is designed. Further, the relationship between the proposed controller and the controlled system is shown.

The PD controller is given as follows.

$$\begin{bmatrix} T_1 \\ T_5 \end{bmatrix} = \begin{bmatrix} K_{P1} & 0 \\ 0 & K_{P5} \end{bmatrix} \begin{bmatrix} \alpha_1(t) - \alpha_{d1}(t) \\ \alpha_5(t) - \alpha_{d5}(t) \end{bmatrix} + \begin{bmatrix} K_{D1} & 0 \\ 0 & K_{D5} \end{bmatrix} \begin{bmatrix} \dot{\alpha}_1(t) - \dot{\alpha}_{d1}(t) \\ \dot{\alpha}_5(t) - \dot{\alpha}_{d5}(t) \end{bmatrix} \quad (8)$$

where  $K_{P_i} > 0$ ,  $K_{D_i} > 0$ , and  $\alpha_{di}$  is the desired trajectory of  $\alpha_i$ . The PD controller guarantees local stability and the required tracking accuracy over a time interval of  $[0, T]$ . The design conditions of the parameters of the controller

are summarized as follows.

- 1) Choose the design parameters  $\delta_1$  and  $\beta$  such that

$$\begin{aligned} \|q(0) - q_d(0)\| &\leq \delta_1 \beta \\ \|\dot{q}(0) - \dot{q}_d(0)\| &\leq \beta \sqrt{\frac{\|M\| + (\delta_1 + T)^2 \|K_P\|}{\min(M)}}, \quad t \in [0, T]. \end{aligned}$$

- 2) Select a positive definite  $R_0$  such that

$$\begin{aligned} \frac{1}{2} \dot{e}^T (\dot{e}^T \frac{\partial M}{\partial q})^T \dot{q}_d &\leq \dot{e}^T F \dot{e} \\ R_0 - B_0 + (\|D\|/\beta)I &\leq K_D \end{aligned} \quad (9)$$

where

$$\begin{aligned} D &= -M\ddot{q}_d - (\dot{q}_d^T \frac{\partial M}{\partial q})^T \dot{q}_d + \frac{1}{2} \dot{q}_d^T \frac{\partial M}{\partial q} \dot{q}_d - \frac{\partial P_e}{\partial q} \\ K_D &= \begin{bmatrix} K_{D1} & 0 \\ 0 & K_{D5} \end{bmatrix} \quad K_P = \begin{bmatrix} K_{P1} & 0 \\ 0 & K_{P5} \end{bmatrix} \\ M &= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad e = \begin{bmatrix} \alpha_1(t) - \alpha_{d1}(t) \\ \alpha_5(t) - \alpha_{d5}(t) \end{bmatrix} \\ q_d &= \begin{bmatrix} \alpha_{d1}(t) \\ \alpha_{d5}(t) \end{bmatrix}. \end{aligned} \quad (10)$$

The proposed controller yields local stability of the whole system by the following brief explanation.

Let  $V$  be a Lyapunov function

$$\begin{aligned} V(q - q_d, \dot{q} - \dot{q}_d) &= [(\dot{q} - \dot{q}_d)^T M (\dot{q} - \dot{q}_d) + \\ &\quad (q - q_d)^T K_P (q - q_d)] / 2. \end{aligned}$$

The deviation of the function  $V$  along the trajectory of the seven bar dynamics presented by (5) and (10) is obtained as follows<sup>[6]</sup>.

$$\begin{aligned} \dot{V}(x, \dot{x}) &= \dot{x}^T M \ddot{x} + \dot{x}^T \dot{M} \dot{x} / 2 + \dot{x}^T K_P x = \\ &\quad (-\dot{x}^T \dot{M} \dot{x} - \dot{x}^T (\dot{x}^T N \dot{x}) - \dot{x}^T (\dot{x}^T N)^T \dot{q}_d) / 2 - \\ &\quad \dot{x}^T (B_O + K_D) \dot{x} + \dot{x}^T d_O \end{aligned} \quad (11)$$

where

$$x = q - q_d, \quad N = \frac{\partial M}{\partial q}$$

and  $N$  is  $2 \times 2$  inertia tensor and  $B_0 = - \begin{bmatrix} T_{output} \frac{\partial \alpha_4}{\partial \alpha_1} \\ T_{output} \frac{\partial \alpha_4}{\partial \alpha_5} \end{bmatrix}$ .

The first term of (11) becomes zero because

$$\begin{aligned} \dot{x}^T \dot{M} \dot{x} &= \dot{q}^T \frac{\partial \dot{x}^T M \dot{x}}{\partial q} = (\dot{x} + \dot{q}_d)^T (\dot{x}^T N \dot{x}) = \\ &\quad \dot{x}^T (\dot{x}^T N \dot{x}) + \dot{q}_d^T (\dot{x}^T N \dot{x}) = \\ &\quad \dot{x}^T (\dot{x}^T N \dot{x}) + \dot{x}^T (\dot{x}^T N)^T \dot{q}_d. \end{aligned} \quad (12)$$

Then, (11) can be rewritten as

$$\dot{V}(x, \dot{x}) = -\dot{x}^T (B_O + K_D) \dot{x} + \dot{x}^T d_O \quad (13)$$

where the first term of the right hand side is non-positive. In the following, we discuss the second term. Similar to (12), we have that

$$\dot{x}^T M \dot{q}_d = (\dot{x} + \dot{q}_d)^T (\dot{x}^T N) \dot{q}_d. \quad (14)$$

As a result, the second term can be rewritten into

$$\begin{aligned} \dot{x}^T d_O &= -\dot{x}^T M \ddot{q}_d - (\dot{x} + \dot{q}_d)^T (\dot{x}^T N) \dot{q}_d + \\ &\quad \frac{1}{2} \dot{x}^T (\dot{x} + \dot{q}_d)^T N \dot{q}_d - \frac{1}{2} \dot{x}^T (\dot{x}^T N)^T \dot{q}_d + \\ &\quad \dot{x}^T B_O \dot{q}_d + \frac{1}{2} \dot{x}^T (\dot{q}_d^T N) \dot{x} - \dot{x}^T (g + D). \end{aligned} \quad (15)$$

Now, if we set the velocity feedback gain matrix  $K_D$  as

$$K_D = W + Z \quad (16)$$

where

$$\begin{aligned} W &= \text{diag}(w_1, w_2, \dots, w_n), \quad w_i > 0, \quad (1 \leq i \leq n) \\ Z &= \text{diag}(z_1, z_2, \dots, z_n), \quad z_i > 0, \quad (1 \leq i \leq n). \end{aligned}$$

Then, using the result in [6], we obtain

$$\begin{aligned} \dot{V}(x, \dot{x}) &= -\dot{x}^T (B_O + K_D) \dot{x} + \dot{x}^T d_O = \\ &= -\dot{x}^T (W \dot{x} - d) - \dot{x}^T (Z + B_O) \dot{x} + \dot{x}^T R \dot{x} \leq \\ &= -\dot{x}^T (W \dot{x} - d) - \dot{x}^T (Z + B_O - R_O) \dot{x} \end{aligned} \quad (17)$$

where  $R_O$  denotes a positive definite matrix satisfying

$$\dot{x}^T R \dot{x} \leq \dot{x}^T R_O \dot{x}. \quad (18)$$

If each diagonal element of the matrix  $Z$  is set sufficiently large, the second term of the right hand side becomes negative. On the other hand, the value of the first term can become both positive and negative depending on the value of  $\dot{x}$ . In the following, we investigate the first term of (17) in detail. Now, note the following inequality:

$$-\dot{x}^T (W \dot{x} - d) \leq -\|\dot{x}\| (w_m \|\dot{x}\| - \|d\|) \quad (19)$$

where  $w_m = \min(w)$ . Further, suppose that

$$p < \|\dot{x}(t)\| \quad (20)$$

at some  $t \in [0, T]$ . If

$$\|d\|/w_m \leq p < \|\dot{x}\| \quad (21)$$

then it follows that

$$-\dot{x}^T (W \dot{x} - d) \leq -\|\dot{x}\| (w_m \|\dot{x}\| - \|d\|) < 0. \quad (22)$$

Therefore, from (17) and (21), we know that if

$$(\|d\|/p)I \leq W, \quad R_O - B_O \leq Z \quad (23)$$

namely,

$$(\|d\|/p)I + R_O - B_O \leq K_D \quad (24)$$

then

$$\dot{V}(x, \dot{x}) < 0. \quad (25)$$

Hence, if  $p < \|\dot{x}(t)\|$  at time  $t \in [0, T]$ , the function  $\dot{V}(x, \dot{x})$  decreases with increasing time  $t \in [0, T]$ . The convergence is ensured for the following bound<sup>[6]</sup>.

$$\|q(t) - q_d(t)\| \leq \beta \sqrt{\frac{\|M\| + (\delta_1 + T)^2 \|K_P\|}{\min(K_P)}}. \quad (26)$$

We summarize the above explanation as follows. When the design conditions (1) and (2) are satisfied, the trajectory tracking can be realized with the required accuracy using (26).

## 4 Simulation results

In this section, we present numerical simulations to demonstrate the effectiveness of the proposed scheme. Simulation studies are conducted by using the structure shown in Fig. 1. The numerical values of the link parameters used in the simulations are shown in Table 1. During the simulation, we set  $T_{\text{output}} = 0$ ,  $f_0 = 10$  mm,  $f_1 = 180$  mm, and  $f_2 = 50$  mm, the constant velocity motor and the servomotor were instructed to complete the motions  $\alpha_{d1} = \pi/8 + 1.6t$ , and  $\alpha_{d5} = 2\pi/3 + 0.25\sin(2\pi/5)/t$ . The desired non-uniform motion can be obtained by combining the above two motions. Here, the mass centers are located at the mid-points of the links.

Table 1 Design parameters of the hybrid linkage system

Link	Mass (g)	Dimensions (mm)
$L_1$	388.5	35.0 mm
$L_5$	284	39.5 mm
$L_6$	70	69.1 mm
$L_4$	86	170.0 mm
$L_3$	71	153.0 mm
$L_2$	62.9 ( $L_2$ )	55.84 mm
$L_7$	70.0 ( $L_7$ )	76.57 mm
		57.2°

Considering the design conditions for the PD controller in the previous section, the parameters of the controller were designed as follows:

$$K_{P1} = 0.8, \quad K_{D1} = 0.01, \quad K_{P5} = 2, \quad K_{D5} = 0.05$$

where  $q_d(0) = [\pi/8, 2\pi/3]^T$ , and  $\dot{q}_d(0) = [1.6, \pi/10]^T$ . Figs. 2 and 3 show the real and the desired outputs of  $\alpha_1$  and  $\alpha_5$ . The results indicate that the outputs of the constant velocity motor and servomotor are obtained. It is known that the tracking performance can be improved by selecting the parameters  $K_{P_i}$  and  $K_{D_i}$  satisfying (9) and (26). In the simulation, to avoid the singularity, we set that  $\alpha_3 - \alpha_4$  and  $\alpha_6 - \alpha_7$  were not less than  $\pi/90$  (rad). Also, to obtain the desired linkage position, we set  $\alpha_i - \alpha_{i0} < \pi/2$ . It can be seen that the nonlinear output motion is obtained from the reference position  $\alpha_1$  parallel and having the same direction

with  $x$  coordinate (see Fig. 4). The slider output is shown in Fig. 5.

**Remark.** It is worth mentioning that parameters  $K_{P_i}$  and  $K_{D_i}$  are selected by a priori trial and error, and that the design conditions 1) and 2) are satisfied by choosing parameters  $\delta_1$ ,  $R_0$ , and  $\beta$ . As a result, (26) is satisfied. However, the control performance is limited by parameters  $\delta_1$ ,  $R_0$ , and  $\beta$ . Therefore, to improve the control performance, we need to design  $K_{P_i}$  and  $K_{D_i}$  by a priori trial and error so that the desired  $q(t) - q_d(t)$  is obtained provided that the fitting parameters  $\delta_1$ ,  $R_0$ , and  $\beta$  are selected.

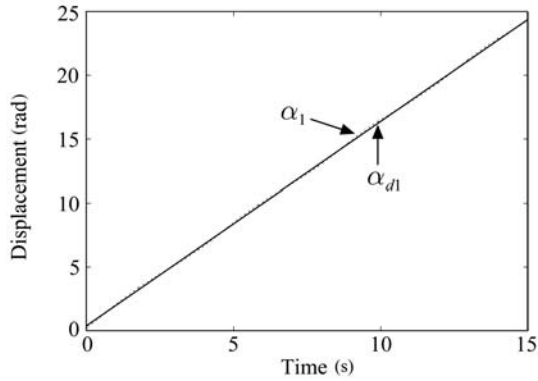


Fig. 2 Output of  $\alpha_1$

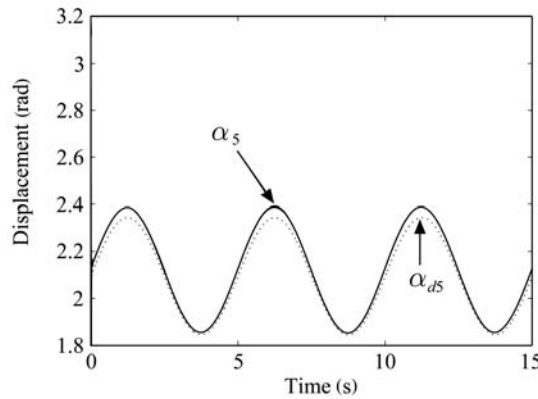


Fig. 3 Output of  $\alpha_5$

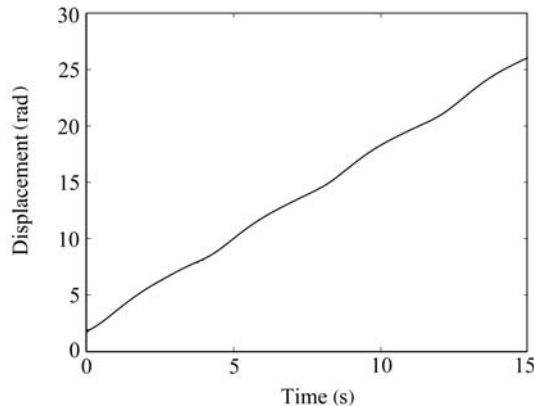


Fig. 4 Non-uniform motion of the system

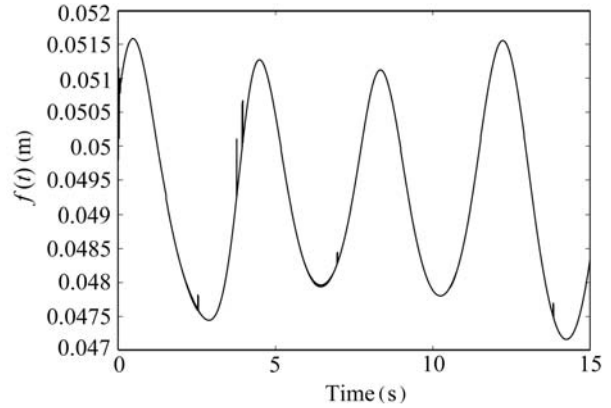


Fig. 5 The slider output

## 5 Conclusions

The Hamiltonian dynamics model of a hybrid system with the capability of producing a non-uniform motion was given. A decentralized PD controller design for the Hamiltonian hybrid system was considered. A relationship between the system parameters and the proposed decentralized PD controller parameters was shown to ensure local stability and tracking performance. Simulation results are presented to examine the proposed method in this paper.

## References

- [1] W. Bradshaw. Control of Hybrid Machines, Ph.D. dissertation, Liverpool John Moores University, UK, 1997.
- [2] Z. Yuan. Design and Control of Hybrid Machines, Ph.D. dissertation, Liverpool John Moores University, UK, 2000.
- [3] H. Yu, M. Deng, M. J. Gilmartin, T. C. Yang. Full Dynamic Model of a Hybrid Seven-bar System. In *Proceedings of the 3rd World Manufacturing Congress*, New York, USA, CD-ROM, 2001.
- [4] M. Deng, H. Yu, M. J. Gilmartin, T. C. Yang. Lagrangian Dynamics and Analysis of a Hybrid Linkage System. *International Journal of Computers, Systems and Signals*, vol. 2, no. 1, pp. 54–71, 2001.
- [5] R. Ortega, A. van der Schaft, B. Maschke, G. Escobar. Interconnection and Damping Assignment Passivity-based Control of Port-controlled Hamiltonian Systems. *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.
- [6] S. Kawamura, F. Miyazaki, S. Arimoto. Is a Local Linear PD Feedback Control Law Effective for Trajectory Tracking of Robot Motion? In *Proceedings of IEEE International Conference of Robotics Automation*, IEEE Press, Philadelphia, USA, vol. 3, pp. 1335–1340, 1988.
- [7] M. Liu. Decentralized Control of Robot Manipulators: Non-linear and Adaptive Approaches. *IEEE Transactions on Automatic Control*, vol. 44, no. 2, pp. 357–363, 1999.
- [8] H. Yu, S. Lloyd. Variable Structure Adaptive Control of Robot Manipulators. *IEE Proceedings of Control Theory Application*, vol. 144, no. 2, pp. 167–176, 1997.

- [9] L. Sciavicco, B. Siciliano. *Modelling and Control of Robot Manipulator*, Springer, London, 2000.
- [10] C. Su, Y. Stepanenko. Hybrid Adaptive/Robust Motion Control of Rigid Electrically-driven Robot Manipulators. *IEEE Transactions on Robotics and Automation*, vol. 11, no. 3, pp. 426–432, 1995.
- [11] J. McCarthy. *Geometric Design of Linkages*, Springer, New York, USA, 2000.



**Mingcong Deng** received his B.Sc. and M.Sc. degrees in control engineering from Northeastern University, PRC, in 1986 and 1991, respectively, and his Ph.D. degree in systems science from Kumamoto University, Japan, in 1997. From 1997 to 2000 he was with Kumamoto University. From 2000 to 2001 he was with University of Exeter, UK, and then spent one year at the NTT

Communication Science Laboratories for human arm dynamics research. From the end of 2002, he has been working at the Department of Systems Engineering, Okayama University, where he is currently an associate professor. He is a member of SICE, IEICE, JSME, ICROS, and IEEE. His research interests include living body measurement, nonlinear system modeling and control (including operator-based control), strong stability-based control, robust parallel compensation, and fault diagnosis.



**Hongnian Yu** received his Ph.D. degree in robotics at King's College London, UK, in 1990–1994. He was a lecturer in control and systems engineering at Yanshan University, PRC, in 1985–1990, a research fellow in manufacturing systems at Sussex University, UK, in 1994–1996, a lecturer in

artificial intelligence at Liverpool John Moore's University, UK, in 1996–1999, a lecturer in control and systems engineering at the University of Exeter, UK, in 1999–2002, and a senior lecturer in computing at the University of Bradford, UK, in 2002–2004. Currently, he is professor of computer science and head of Mobile Computing and Distributed Control Systems Research Group at Staffordshire University, UK. He has published over 100 research papers. He is an EPSRC college member, a member of IEEE, and a committee member of several conferences and journal editorial boards. His research interests include experience in neural networks, mobile computing, modelling, control of robot manipulators, and modelling, scheduling, planning, and simulations of large discrete event dynamic systems with applications to manufacturing systems, supply chains, transportation networks, and computer networks.



**Akira Inoue** received his B.Sc. and M.Sc., and Ph.D. engineering degrees in applied mathematics and physics from Kyoto University, Japan, in 1966, 1968, and 1977, respectively. From 1977 to 1978, he was with the University of Alberta, Canada, and from 1978 to 1987 he was with Kumamoto University, Japan. From 1987 to 1995, he was a professor at the Department of Information Technology and Department of Systems Engineering, Okayama University, Japan. He is a fellow of the Society of Instrument and Control Engineers (SICE) and a member of Society for Industrial and Applied Mathematics (SIAM) and IEEE. He was chairperson of the organizing committee for *Japan Joint Automatic Control Conference*, 2003, and was general chair of *SICE Annual Conference*, 2005. His research interests include adaptive control, nonlinear control of mechanical systems, and model predictive control.